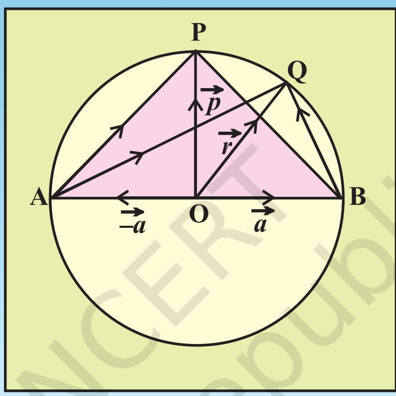
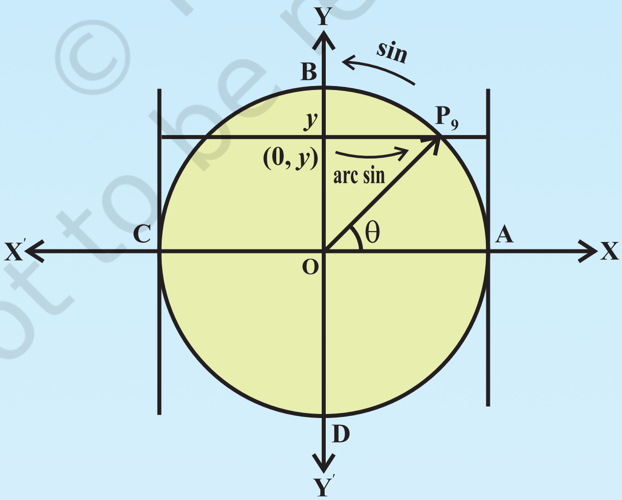
Activities for
Class XII





The basic principles of learning mathematics are : (a) learning should be related to each child individually (b) the need for mathematics should develop from an intimate acquaintance with the environment (c) the child should be active and interested, (d) concrete material and wide variety of illustrations are needed to aid the learning process (e) understanding should be encouraged at each stage of acquiring a particular skill (f) content should be broadly based with adequate appreciation of the links between the various branches of mathematics, (g) correct mathematical usage should be encouraged at all stages.

– Ronwill

Activity 1

Objective

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \perp m\}$ is symmetric but neither reflexive nor transitive.

Material Required

A piece of plywood, some pieces of wires (8), nails, white paper, glue etc.

Method of Construction

Take a piece of plywood and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig.1.

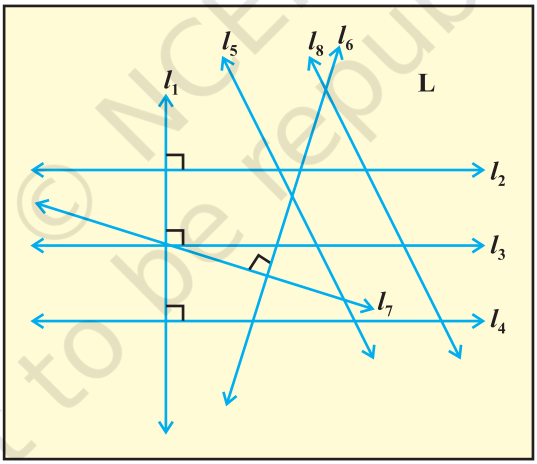


Fig. 1

Demonstration

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4. [see Fig. 1]

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_1, l_2), (l_1, l_3), (l_1, l_4), (l_6, l_7) \in R$

OBSERVATION

1. In Fig. 1, no line is perpendicular to itself, so the relation $R = \{(l, m) : l \perp m\}$ ______ reflexive (is/is not).

2. In Fig. 1, $l_1 \perp l_2$. Is $l_2 \perp l_1$? ______ (Yes/No)

   $\therefore$ $(l_1, l_2) \in R \Rightarrow (l_2, l_1)$ ______ R $(\notin / \in)$

   Similarly, $l_3 \perp l_1$. Is $l_1 \perp l_3$? ______ (Yes/No)

   $\therefore$ $(l_3, l_1) \in R \Rightarrow (l_1, l_3)$ ______ R $(\notin / \in)$

   Also, $l_6 \perp l_7$. Is $l_7 \perp l_6$? ______ (Yes/No)

   $\therefore$ $(l_6, l_7) \in R \Rightarrow (l_7, l_6)$ ______ R $(\notin / \in)$

   $\therefore$ The relation R .... symmetric (is/is not)

3. In Fig. 1, $l_2 \perp l_1$ and $l_1 \perp l_3$. Is $l_2 \perp l_3$? ... (Yes/No)

   i.e., $(l_2, l_1) \in R$ and $(l_1, l_3) \in R \Rightarrow (l_2, l_3)$ ______ R $(\notin / \in)$

   $\therefore$ The relation R .... transitive (is/is not).

APPLICATION

This activity can be used to check whether a given relation is an equivalence relation or not.

NOTE

1. In this case, the relation is not an equivalence relation.
2. The activity can be repeated by taking some more wire in different positions.

Activity 2

Objective

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalence relation.

Material Required

A piece of plywood, some pieces of wire (8), plywood, nails, white paper, glue.

Method of Construction

Take a piece of plywood of convenient size and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig. 2.

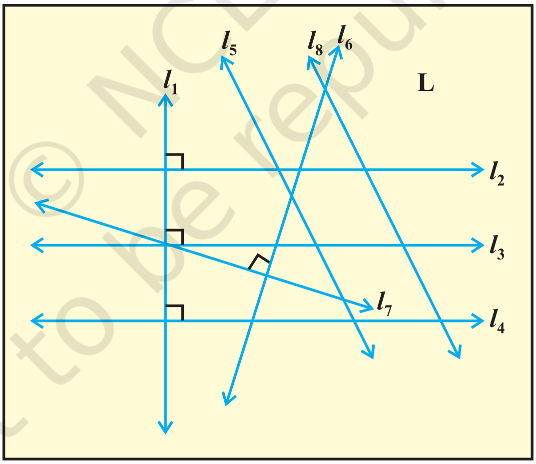


Fig. 2

Demonstration

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4 (see Fig. 2).

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_2, l_3), (l_3, l_4), (l_5, l_8), \in$ R

OBSERVATION

1. In Fig. 2, every line is parallel to itself. So the relation R = $\{(l, m) : l \parallel m\}$ .... reflexive relation (is/is not)

2. In Fig. 2, observe that $l_2 \parallel l_3$. Is l_3 ... l_2? ($\nparallel$ / $\parallel$)

   So, $(l_2, l_3) \in$ R $\Rightarrow (l_3, l_2)$ ... R ($\notin$ / $\in$)

   Similarly, $l_3 \parallel l_4$. Is l_4 ...l_3? ($\nparallel$ / $\parallel$)

   So, $(l_3, l_4) \in$ R $\Rightarrow (l_4, l_3)$ ... R ($\notin$ / $\in$)

   and $(l_5, l_8) \in$ R $\Rightarrow (l_8, l_5)$ ... R ($\notin$ / $\in$)

$\therefore$ The relation R ... symmetric relation (is/is not)

3. In Fig. 2, observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Is l_2 ... l_4 ? ($\parallel$ / $\parallel$)

   So, $(l_2, l_3) \in$ R and $(l_3, l_4) \in$ R $\Rightarrow (l_2, l_4)$ ... R ($\in$ / $\notin$)

   Similarly, $l_3 \parallel l_4$ and $l_4 \parallel l_2$. Is l_3 ... l_2 ? ($\nparallel$ / $\parallel$)

   So, $(l_3, l_4) \in$ R, $(l_4, l_2) \in$ R $\Rightarrow (l_3, l_2)$ ... R ($\in$, $\notin$)

Thus, the relation R ... transitive relation (is/is not)

Hence, the relation R is reflexive, symmetric and transitive. So, R is an equivalence relation.

APPLICATION

This activity is useful in understanding the concept of an equivalence relation.

NOTE

This activity can be repeated by taking some more wires in different positions.

Activity 3

OBJECTIVE

To demonstrate a function which is not one-one but is onto.

MATERIAL REQUIRED

Cardboard, nails, strings, adhesive and plastic strips.

METHOD OF CONSTRUCTION

1. Paste a plastic strip on the left hand side of the cardboard and fix three nails on it as shown in the Fig.3.1. Name the nails on the strip as 1, 2 and 3.
2. Paste another strip on the right hand side of the cardboard and fix two nails in the plastic strip as shown in Fig.3.2. Name the nails on the strip as *a* and *b*.
3. Join nails on the left strip to the nails on the right strip as shown in Fig. 3.3.

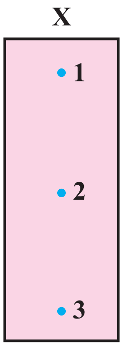


Fig. 3.1

Fig. 3.2

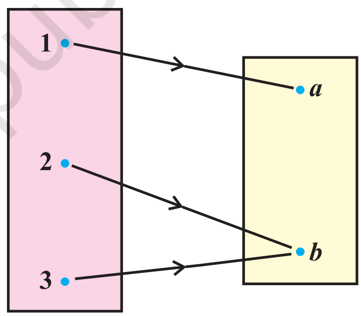


Fig. 3.3

DEMONSTRATION

1. Take the set X = {1, 2, 3}
2. Take the set Y = {*a*, *b*}
3. Join (correspondence) elements of X to the elements of Y as shown in Fig. 3.3

OBSERVATION

1. The image of the element 1 of X in Y is __________.

   The image of the element 2 of X in Y is __________.

The image of the element 3 of X in Y is __________.

So, Fig. 3.3 represents a __________ .

2. Every element in X has a _________ image in Y. So, the function is _________(one-one/not one-one).

3. The pre-image of each element of Y in X _________ (exists/does not exist). So, the function is ________ (onto/not onto).

Application

This activity can be used to demonstrate the concept of one-one and onto function.

Note

Demonstrate the same activity by changing the number of the elements of the sets X and Y.

Activity 4

OBJECTIVE

To demonstrate a function which is one-one but not onto.

MATERIAL REQUIRED

Cardboard, nails, strings, adhesive and plastic strips.

METHOD OF CONSTRUCTION

1. Paste a plastic strip on the left hand side of the cardboard and fix two nails in it as shown in the Fig. 4.1. Name the nails as *a* and *b*.
2. Paste another strip on the right hand side of the cardboard and fix three nails on it as shown in the Fig. 4.2. Name the nails on the right strip as 1, 2 and 3.
3. Join nails on the left strip to the nails on the right strip as shown in the Fig. 4.3.

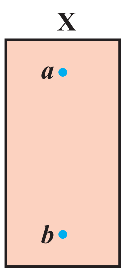


Fig. 4.1

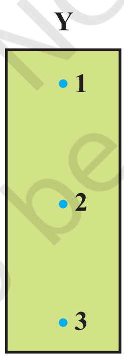


Fig. 4.2

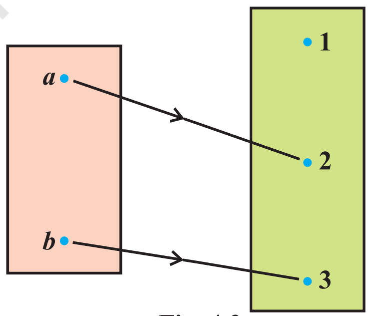


Fig. 4.3

DEMONSTRATION

1. Take the set X = {*a*, *b*}
2. Take the set Y = {1, 2, 3}.
3. Join elements of X to the elements of Y as shown in Fig. 4.3.

OBSERVATION

1. The image of the element *a* of X in Y is ______________.

   The image of the element *b* of X in Y is ______________.

   So, the Fig. 4.3 represents a _____________________.

2. Every element in X has a _________ image in Y. So, the function is _____________ (one-one/not one-one).

3. The pre-image of the element 1 of Y in X __________ (exists/does not exist). So, the function is __________ (onto/not onto).

   Thus, Fig. 4.3 represents a function which is _________ but not onto.

APPLICATION

This activity can be used to demonstrate the concept of one-one but not onto function.

Activity 5

Objective

To draw the graph of $\sin^{-1} x$, using the graph of sin x and demonstrate the concept of mirror reflection (about the line $y = x$).

Material Required

Cardboard, white chart paper, ruler, coloured pens, adhesive, pencil, eraser, cutter, nails and thin wires.

Method of Construction

1. Take a cardboard of suitable dimensions, say, 30 cm × 30 cm.
2. On the cardboard, paste a white chart paper of size 25 cm × 25 cm (say).
3. On the paper, draw two lines, perpendicular to each other and name them X′OX and YOY′ as rectangular axes [see Fig. 5].

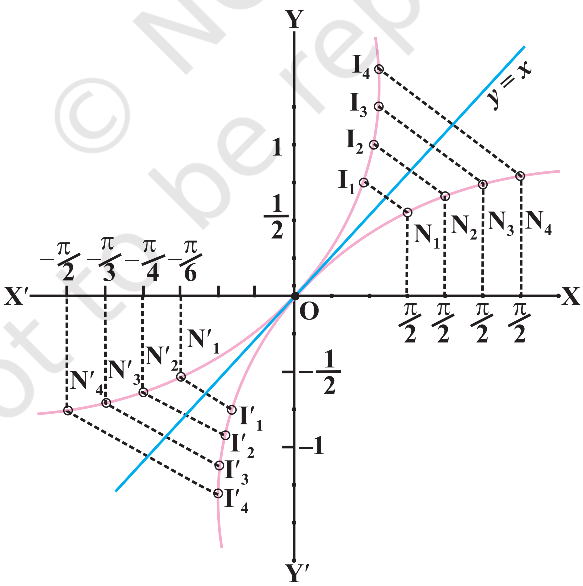


Fig. 5

4. Graduate the axes approximately as shown in Fig. 5.1 by taking unit on X-axis = 1.25 times the unit of Y-axis.

5. Mark approximately the points

$\left(\frac{\pi}{6}, \sin\frac{\pi}{6}\right), \left(\frac{\pi}{4}, \sin\frac{\pi}{4}\right), \ldots, \left(\frac{\pi}{2}, \sin\frac{\pi}{2}\right)$ in the coordinate plane and at each point fix a nail.

6. Repeat the above process on the other side of the *x*-axis, marking the points $\left(\frac{-\pi}{6}, \sin\frac{-\pi}{6}\right), \left(\frac{-\pi}{4}, \sin\frac{-\pi}{4}\right), \ldots, \left(\frac{-\pi}{2}, \sin\frac{-\pi}{2}\right)$ approximately and fix nails on these points as N_1', N_2', N_3', N_4'. Also fix a nail at O.

7. Join the nails with the help of a tight wire on both sides of *x*-axis to get the graph of sin *x* from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.

8. Draw the graph of the line $y = x$ (by plotting the points (1,1), (2, 2), (3, 3), ... etc. and fixing a wire on these points).

9. From the nails N_1, N_2, N_3, N_4, draw perpendicular on the line $y = x$ and produce these lines such that length of perpendicular on both sides of the line $y = x$ are equal. At these points fix nails, I_1, I_2, I_3, I_4.

10. Repeat the above activity on the other side of X- axis and fix nails at I_1', I_2', I_3', I_4'.

11. Join the nails on both sides of the line $y = x$ by a tight wire that will show the graph of $y = \sin^{-1} x$.

Demonstration

Put a mirror on the line $y = x$. The image of the graph of sin *x* in the mirror will represent the graph of $\sin^{-1} x$ showing that $\sin^{-1} x$ is mirror reflection of sin *x* and vice versa.

OBSERVATION

The image of point N_1 in the mirror (the line $y = x$) is _________.

The image of point N_2 in the mirror (the line $y = x$) is _________.

The image of point N_3 in the mirror (the line $y = x$) is _________.

The image of point N_4 in the mirror (the line $y = x$) is _________.

The image of point N'_1 in the mirror (the line $y = x$) is _________.

The image point of N'_2 in the mirror (the line $y = x$) is _________.

The image point of N'_3 in the mirror (the line $y = x$) is _________.

The image point of N'_4 in the mirror (the line $y = x$) is _________.

The image of the graph of six x in $y = x$ is the graph of _________, and the image of the graph of $\sin^{-1}x$ in $y = x$ is the graph of __________.

APPLICATION

Similar activity can be performed for drawing the graphs of $\cos^{-1}x$, $\tan^{-1}x$, etc.

Activity 6

Objective

To explore the principal value of the function $\sin^{-1}x$ using a unit circle.

Material Required

Cardboard, white chart paper, rails, ruler, adhesive, steel wires and needle.

Method of Construction

1. Take a cardboard of a convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on it.
3. Through the centre of the circle, draw two perpendicular lines X′OX and YOY′ representing x-axis and y-axis, respectively as shown in Fig. 6.1.
4. Mark the points A, C, B and D, where the circle cuts the x-axis and y-axis, respectively as shown in Fig. 6.1.
5. Fix two rails on opposite sides of the cardboard which are parallel to y-axis. Fix one steel wire between the rails such that the wire can be moved parallel to x-axis as shown in Fig. 6.2.

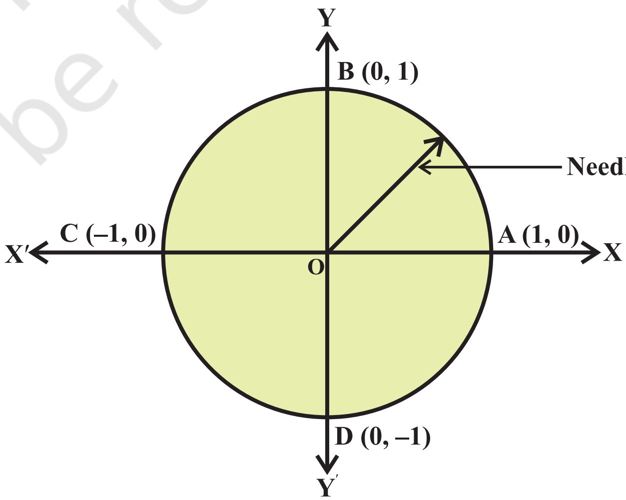


Fig. 6.1

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle Fig. 6.2.

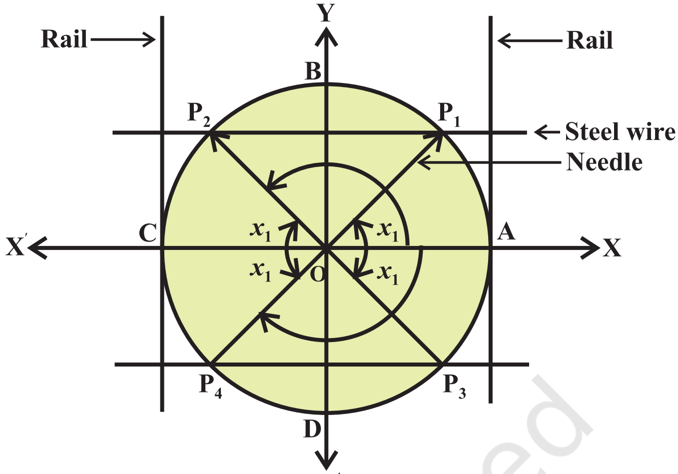


Fig. 6.2

Demonstration

1. Keep the needle at an arbitrary angle, say x_1 with the positive direction of x-axis. Measure of angle in radian is equal to the length of intercepted arc of the unit circle.

2. Slide the steel wire between the rails, parallel to x-axis such that the wire meets with free end of the needle (say P_1) (Fig. 6.2).

3. Denote the y-coordinate of the point P_1 as y_1, where y_1 is the perpendicular distance of steel wire from the x-axis of the unit circle giving $y_1 = \sin x_1$.

4. Rotate the needle further anticlockwise and keep it at the angle $\pi - x_1$. Find the value of y-coordinate of intersecting point P_2 with the help of sliding steel wire. Value of y-coordinate for the points P_1 and P_2 are same for the different value of angles, $y_1 = \sin x_1$ and $y_1 = \sin (\pi - x_1)$. This demonstrates that sine function is not one-to-one for angles considered in first and second quadrants.

5. Keep the needle at angles $-x_1$ and $(-\pi + x_1)$, respectively. By sliding down the steel wire parallel to x-axis, demonstrate that y-coordinate for the points P_3 and P_4 are the same and thus sine function is not one-to-one for points considered in 3rd and 4th quadrants as shown in Fig. 6.2.

6. However, the y-coordinate of the points P_3 and P_1 are different. Move the needle in anticlockwise direction starting from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ and look at the behaviour of y-coordinates of points P_5, P_6, P_7 and P_8 by sliding the steel wire parallel to x-axis accordingly. y-coordinate of points P_5, P_6, P_7 and P_8 are different (see Fig. 6.3). Hence, sine function is one-to-one in the domian $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and its range lies between – 1 and 1.

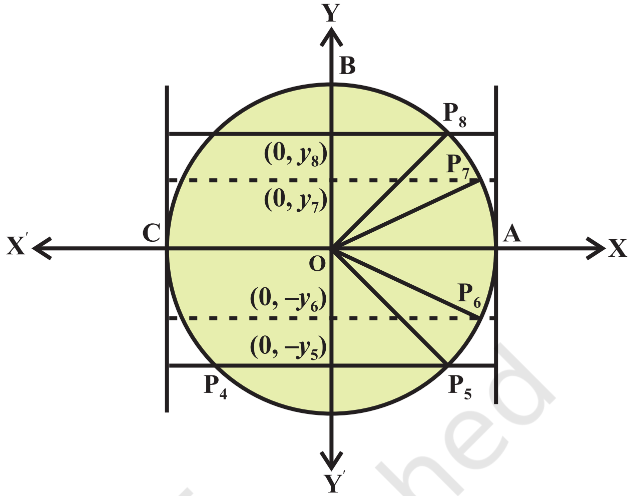


Fig. 6.3

7. Keep the needle at any arbitrary angle say θ lying in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and denote the y-coordinate of the intersecting point P_9 as y. (see Fig. 6.4). Then $y = \sin\theta$ or $\theta = \text{arc} \sin^{-1}y$) as sine function is one-one and onto in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and range [–1, 1]. So, its inverse arc sine function exist. The domain of arc sine function is [–1, 1] and

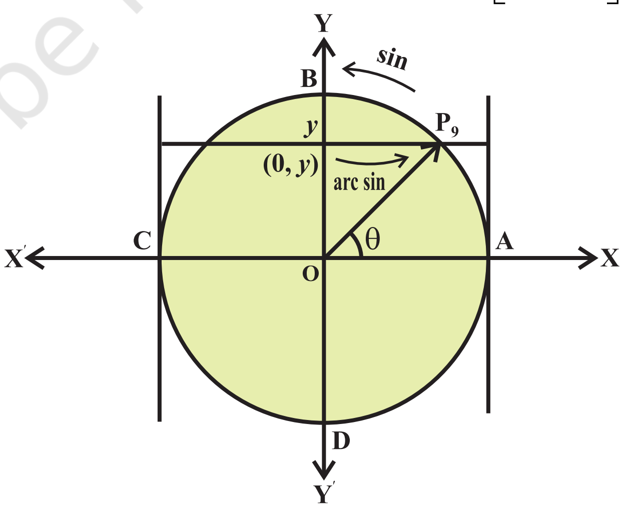


Fig. 6.4

range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. This range is called the principal value of arc sine function (or $\sin^{-1}$ function).

OBSERVATION

1. sine function is non-negative in _________ and __________ quadrants.
2. For the quadrants 3rd and 4th, sine function is _________.
3. $\theta = \text{arc} \sin y \Rightarrow y =$ ________ θ where $-\frac{\pi}{2} \le \theta \le$ ________.
4. The other domains of sine function on which it is one-one and onto provides _________ for arc sine function.

APPLICATION

This activity can be used for finding the principal value of arc cosine function ($\cos^{-1} y$).

Activity 7

OBJECTIVE

To sketch the graphs of a^x and $\log_a x$, $a > 0$, $a \neq 1$ and to examine that they are mirror images of each other.

MATERIAL REQUIRED

Drawing board, geometrical instruments, drawing pins, thin wires, sketch pens, thick white paper, adhesive, pencil, eraser, a plane mirror, squared paper.

METHOD OF CONSTRUCTION

1. On the drawing board, fix a thick paper sheet of convenient size 20 cm × 20 cm (say) with adhesive.

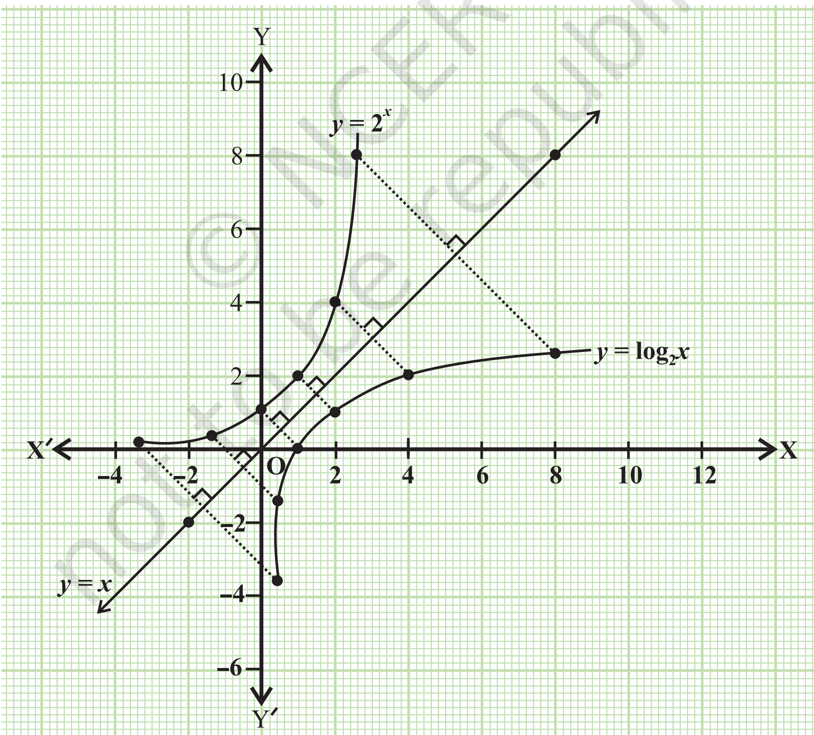


Fig. 7

2. On the sheet, take two perpendicular lines XOX′ and YOY′, depicting coordinate axes.

3. Mark graduations on the two axes as shown in the Fig. 7.

4. Find some ordered pairs satisfying $y = a^x$ and $y = \log_a x$. Plot these points corresponding to the ordered pairs and join them by free hand curves in both the cases. Fix thin wires along these curves using drawing pins.

5. Draw the graph of $y = x$, and fix a wire along the graph, using drawing pins.

Demonstration

1. For a^x, take $a = 2$ (say), and find ordered pairs satisfying it as

x	0	1	–1	2	–2	3	–3	$\frac{1}{2}$	$-\frac{1}{2}$	4
2^x	1	2	0.5	4	$\frac{1}{4}$	8	$\frac{1}{8}$	1.4	0.7	16

and plot these ordered pairs on the squared paper and fix a drawing pin at each point.

2. Join the bases of drawing pins with a thin wire. This will represent the graph of 2^x.

3. $\log_2 x = y$ gives $x = 2^y$. Some ordered pairs satisfying it are:

x	1	2	$\frac{1}{2}$	4	$\frac{1}{4}$	8	$\frac{1}{8}$
y	0	1	–1	2	–2	3	–3

Plot these ordered pairs on the squared paper (graph paper) and fix a drawing pin at each plotted point. Join the bases of the drawing pins with a thin wire. This will represent the graph of $\log_2 x$.

4. Draw the graph of line $y = x$ on the sheet.
5. Place a mirror along the wire representing $y = x$. It can be seen that the two graphs of the given functions are mirror images of each other in the line $y = x$.

Observation

1. Image of ordered pair (1, 2) on the graph of $y = 2^x$ in $y = x$ is ______. It lies on the graph of y = _______.
2. Image of the point (4, 2) on the graph $y = \log_2 x$ in $y = x$ is _________ which lies on the graph of y = _______.

Repeat this process for some more points lying on the two graphs.

Application

This activity is useful in understanding the concept of (exponential and logarithmic functions) which are mirror images of each other in $y = x$.

Activity 8

OBJECTIVE

To establish a relationship between common logarithm (to the base 10) and natural logarithm (to the base e) of the number x.

MATERIAL REQUIRED

Hardboard, white sheet, graph paper, pencil, scale, log tables or calculator (graphic/scientific).

METHOD OF CONSTRUCTION

1. Paste a graph paper on a white sheet and fix the sheet on the hardboard.
2. Find some ordered pairs satisfying the function $y = \log_{10} x$. Using log tables/ calculator and draw the graph of the function on the graph paper (see Fig. 8)

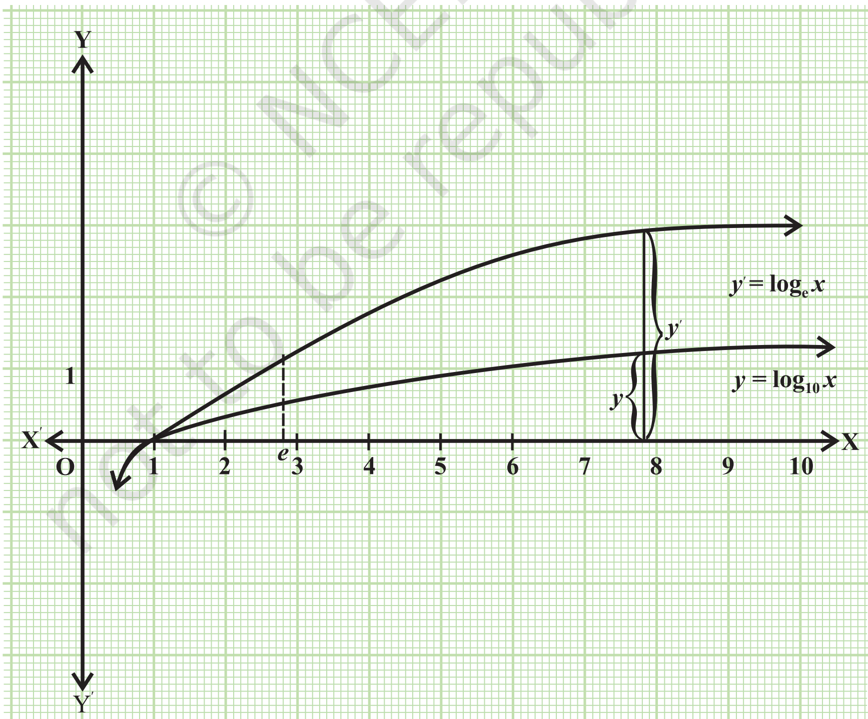


Fig. 8

3. Similarly, draw the graph of $y' = \log_e x$ on the same graph paper as shown in the figure (using log table/calculator).

Demonstration

1. Take any point on the positive direction of x-axis, and note its x-coordinate.
2. For this value of x, find the value of y-coordinates for both the graphs of $y = \log_{10} x$ and $y' = \log_e x$ by actual measurement, using a scale, and record them as y and y', respectively.
3. Find the ratio $\frac{y}{y'}$.
4. Repeat the above steps for some more points on the x-axis (with different values) and find the corresponding ratios of the ordinates as in Step 3.
5. Each of these ratios will nearly be the same and equal to 0.4, which is approximately equal to $\frac{1}{\log_e 10}$.

Observation

S.No.	Points on the x-axis	$y = \log_{10} x$	$y' = \log_e x$	Ratio $\frac{y}{y'}$ (approximate)
1.	x_1= ______	y_1 = ______	y'_1 = ______	___________
2.	x_2=______	y_2 = ______	y'_2 = ______	___________
3.	x_3=______	y_3 = ______	y'_3 = ______	___________
4.	x_4=______	y_4 = ______	y'_4 = ______	___________
5.	x_5=______	y_5 = ______	y'_5 = ______	___________
6.	x_6=______	y_6 = ______	y'_6 = ______	___________

2. The value of $\frac{y}{y'}$ for each point x is equal to __________ approximately.

3. The observed value of $\frac{y}{y'}$ in each case is approximately equal to the value of $\frac{1}{\log_e 10}$. (Yes/No)

4. Therefore, $\log_{10} x = \frac{\quad}{\log_e 10}$.

Application

This activity is useful in converting log of a number in one given base to log of that number in another base.

Note

Let, $y = \log_{10} x$, i.e., $x = 10^y$.

Taking logarithm to base e on both the sides, we get $\log_e x = y \log_e 10$

or $y = \frac{1}{\log_e 10} (\log_e x)$

$\Rightarrow \frac{\log_{10} x}{\log_e x} = \frac{1}{\log_e 10} = 0.434294$ (using log tables/calculator).

Activity 9

OBJECTIVE

To find analytically the limit of a function $f(x)$ at $x = c$ and also to check the continuity of the function at that point.

MATERIAL REQUIRED

Paper, pencil, calculator.

METHOD OF CONSTRUCTION

1. Consider the function given by $f(x)=\begin{cases}\dfrac{x^2-16}{x-4}, & x\neq 4\\ 10, & x=4\end{cases}$
2. Take some points on the left and some points on the right side of c (= 4) which are very near to c.
3. Find the corresponding values of $f(x)$ for each of the points considered in step 2 above.
4. Record the values of points on the left and right side of c as x and the corresponding values of $f(x)$ in a form of a table.

DEMONSTRATION

1. The values of x and $f(x)$ are recorded as follows:

Table 1 : For points on the left of c (= 4).

x	3.9	3.99	3.999	3.9999	3.99999	3.999999	3.9999999
$f(x)$	7.9	7.99	7.999	7.9999	7.99999	7.999999	7.9999999

2. **Table 2**: For points on the right of c (= 4).

x	4.1	4.01	4.001	4.0001	4.00001	4.000001	4.0000001
$f(x)$	8.1	8.01	8.001	8.0001	8.00001	8.000001	8.0000001

Observation

1. The value of $f(x)$ is approaching to ________, as $x \to 4$ from the left.
2. The value of $f(x)$ is approaching to ________, as $x \to 4$ from the right.
3. So, $\lim_{x \to 4^-} f(x)$ = ________ and $\lim_{x \to 4^+} f(x)$ = ________.
4. Therefore, $\lim_{x \to 4} f(x)$ = ________ , $f(4)$ = ________.
5. Is $\lim_{x \to 4} f(x) = f(4)$ ________ ? (Yes/No)
6. Since $f(c) \neq \lim_{x \to c} f(x)$, so, the function is ________ at $x = 4$ (continuous/ not continuous).

Application

This activity is useful in understanding the concept of limit and continuity of a function at a point.

Activity 10

Objective

To verify that for a function *f* to be continuous at given point x_0,

$$\Delta y = \left| f\left(x_0 + \Delta x\right) - f\left(x_0\right) \right| \text{ is}$$

arbitrarily small provided. Δx is sufficiently small.

Material Required

Hardboard, white sheets, pencil, scale, calculator, adhesive.

Method of Construction

1. Paste a white sheet on the hardboard.
2. Draw the curve of the given continuous function as represented in the Fig. 10.
3. Take any point A (x_0, 0) on the positive side of *x*-axis and corresponding to this point, mark the point P (x_0, y_0) on the curve.

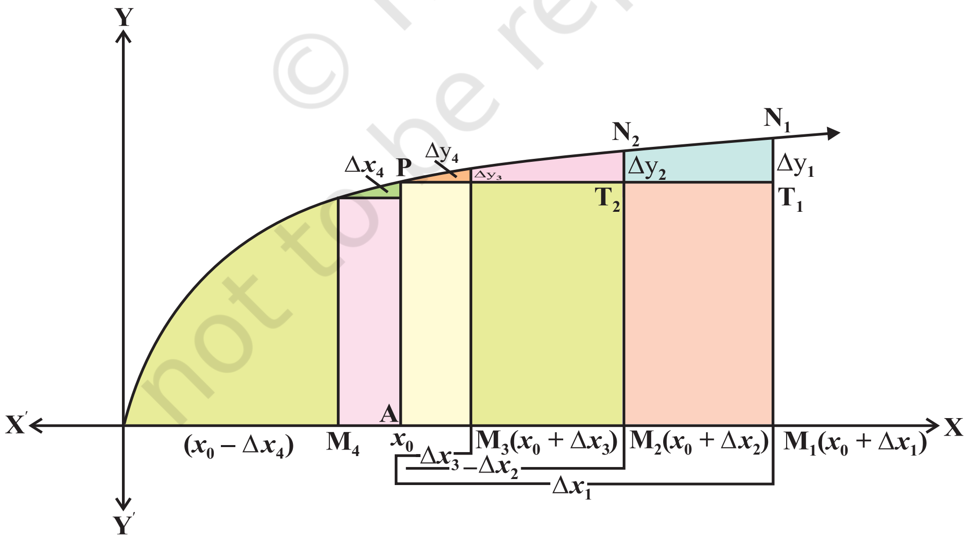


Fig. 10

Demonstration

1. Take one more point M_1 $(x_0 + \Delta x_1, 0)$ to the right of A, where Δx_1 is an increment in x.
2. Draw the perpendicular from M_1 to meet the curve at N_1. Let the coordinates of N_1 be $(x_0 + \Delta x_1, y_0 + \Delta y_1)$
3. Draw a perpendicular from the point P (x_0, y_0) to meet N_1M_1 at T_1.
4. Now measure $AM_1 = \Delta x_1$ (say) and record it and also measure $N_1T_1 = \Delta y_1$ and record it.
5. Reduce the increment in x to Δx_2 (i.e., $\Delta x_2 < \Delta x_1$) to get another point $M_2\left(x_0 + \Delta x_2, 0\right)$. Get the corresponding point N_2 on the curve
6. Let the perpendicular PT_1 intersects N_2M_2 at T_2.
7. Again measure $AM_2 = \Delta x_2$ and record it.

   Measure $N_2T_2 = \Delta y_2$ and record it.
8. Repeat the above steps for some more points so that Δx becomes smaller and smaller.

Observation

S.No.	Value of increment in x_0	Corresponding increment in y
1.	$\lvert\Delta x_1\rvert =$ ________	$\lvert\Delta y_1\rvert =$ ________
2.	$\lvert\Delta x_2\rvert =$ ________	$\lvert\Delta y_2\rvert =$ ________
3.	$\lvert\Delta x_3\rvert =$ ________	$\lvert\Delta y_3\rvert =$ ________
4.	$\lvert\Delta x_4\rvert =$ ________	$\lvert\Delta y_4\rvert =$ ________
5.	$\lvert\Delta x_5\rvert =$ ________	$\lvert\Delta y_5\rvert =$ ________

6.	$\|\Delta x_6\| =$	$\|\Delta y_6\| =$
7.	$\|\Delta x_7\| =$	$\|\Delta y_7\| =$
8.	$\|\Delta x_8\| =$	$\|\Delta y_8\| =$
9.	$\|\Delta x_9\| =$	$\|\Delta y_9\| =$
10.		

2. So, Δy becomes __________ when Δx becomes smaller.

3. Thus $\lim_{\Delta x \to 0} \Delta y = 0$ for a continuous function.

APPLICATION

This activity is helpful in explaining the concept of derivative (left hand or right hand) at any point on the curve corresponding to a function.

Activity 11

OBJECTIVE

To verify Rolle's Theorem.

MATERIAL REQUIRED

A piece of plywood, wires of different lengths, white paper, sketch pen.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent x-axis and y-axis (see Fig. 11).
3. Take a piece of wire of 15 cm length and bend it in the shape of a curve and fix it on the plywood as shown in the figure.

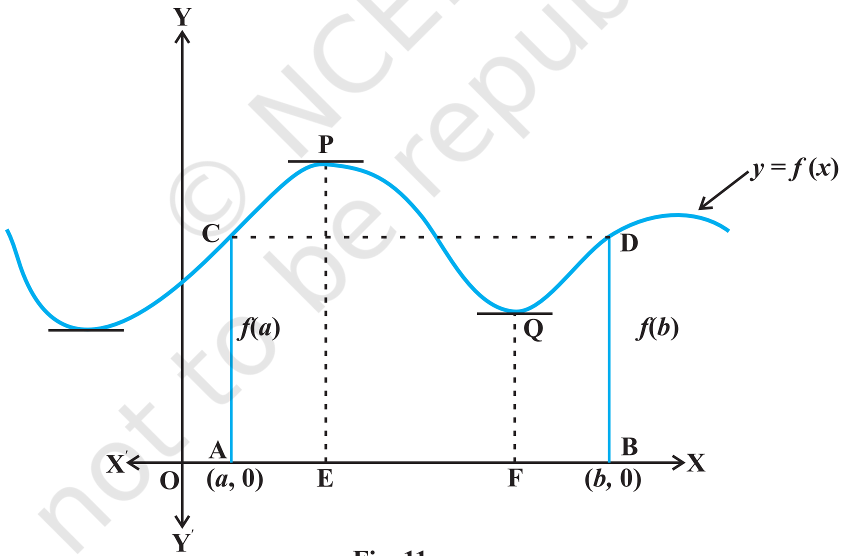


Fig. 11

4. Take two straight wires of the same length and fix them in such way that they are perpendicular to x-axis at the points A and B and meeting the curve at the points C and D (see Fig.11).

Demonstration

1. In the figure, let the curve represent the function $y = f(x)$. Let OA = a units and OB = b units.
2. The coordinates of the points A and B are $(a, 0)$ and $(b, 0)$, respectively.
3. There is no break in the curve in the interval $[a, b]$. So, the function f is continuous on $[a, b]$.
4. The curve is smooth between $x = a$ and $x = b$ which means that at each point, a tangent can be drawn which in turn gives that the function f is differentiable in the interval (a, b).
5. As the wires at A and B are of equal lengths, i.e., AC = BD, so $f(a) = f(b)$.
6. In view of steps (3), (4) and (5), conditions of Rolle's theorem are satisfied. From Fig.11, we observe that tangents at P as well as Q are parallel to x-axis, therefore, $f'(x)$ at P and also at Q are zero.

Thus, there exists at least one value c of x in (a,b) such that $f'(c) = 0$.

Hence, the Rolle's theorem is verified.

Observation

From Fig. 11.

a = _______________, b = ______________

$f(a)$ = ____________, $f(b)$ = _________ Is $f(a) = f(b)$? (Yes/No)

Slope of tangent at P = __________, so, $f(x)$ (at P) =

Application

This theorem may be used to find the roots of an equation.

Activity 12

Objective

To verify Lagrange's Mean Value Theorem.

Material Required

A piece of plywood, wires, white paper, sketch pens, wires.

Method of Construction

1. Take a piece of plywood and paste a white paper on it.
2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent *x*-axis and *y*-axis (see Fig. 12).
3. Take a piece of wire of about 10 cm length and bend it in the shape of a curve as shown in the figure. Fix this curved wire on the white paper pasted on the plywood.

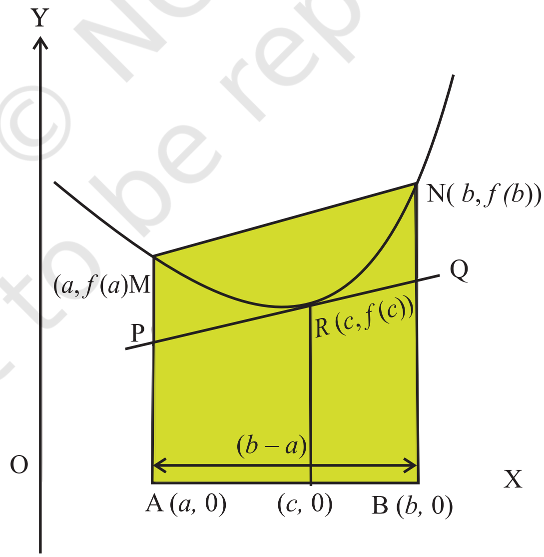


Fig. 12

4. Take two straight wires of lengths 10 cm and 13 cm and fix them at two different points of the curve parallel to y-axis and their feet touching the x-axis. Join the two points, where the two vertical wires meet the curve, using another wire.
5. Take one more wire of a suitable length and fix it in such a way that it is tangential to the curve and is parallel to the wire joining the two points on the curve (see Fig. 12).

DEMONSTRATION

1. Let the curve represent the function $y = f(x)$. In the figure, let OA = a units and OB = b units.
2. The coordinates of A and B are $(a, 0)$ and $(b, 0)$, respectively.
3. MN is a chord joining the points M $(a, f(a))$ and N $(b, f(b))$.
4. PQ represents a tangent to the curve at the point R $(c, f(c))$, in the interval (a, b).
5. $f'(c)$ is the slope of the tangent PQ at $x = c$.
6. $\frac{f(b)-f(a)}{b-a}$ is the slope of the chord MN.
7. MN is parallel to PQ, therefore, $f'(c) = \frac{f(b)-f(a)}{b-a}$. Thus, the Langrange's Mean Value Theorem is verified.

OBSERVATION

1. a = __________, b = ______________,

   $f(a)$ = ________, $f(b)$= ____________.
2. $f(a) - f(b)$ = ________,

   $b - a$ = ________,

3. $\dfrac{f(b)-f(a)}{b-a}$ = ________ = Slope of MN.

4. Since PQ || MN $\Rightarrow$ Slope of PQ $= f'(c) = \dfrac{f(a)-f(a)}{b-a}$.

APPLICATION

Langrange's Mean Value Theorem has significant applications in calculus. For example this theorem is used to explain concavity of the graph.

Activity 13

OBJECTIVE

To understand the concepts of decreasing and increasing functions.

MATERIAL REQUIRED

Pieces of wire of different lengths, piece of plywood of suitable size, white paper, adhesive, geometry box, trigonometric tables.

METHOD OF CONSTRUCTION

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wires of length say 20 cm each and fix them on the white paper to represent *x*-axis and *y*-axis.
3. Take two more pieces of wire each of suitable length and bend them in the shape of curves representing two functions and fix them on the paper as shown in the Fig. 13.

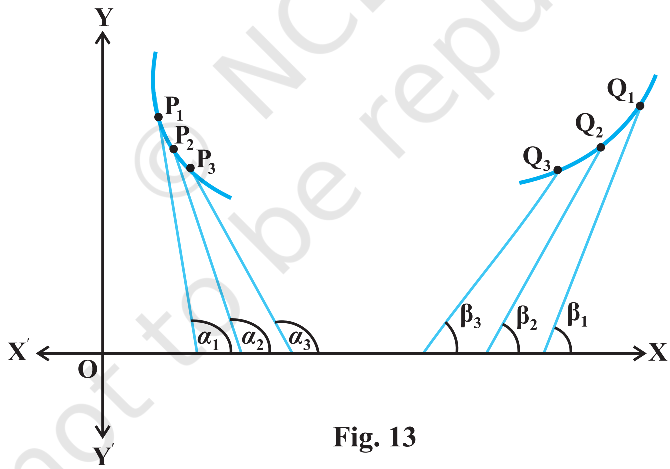


Fig. 13

4. Take two straight wires each of suitable length for the purpose of showing tangents to the curves at different points on them.

DEMONSTRATION

1. Take one straight wire and place it on the curve (on the left) such that it is

tangent to the curve at the point say P_1 and making an angle α_1 with the positive direction of x-axis.

2. α_1 is an obtuse angle, so $\tan\alpha_1$ is negative, i.e., the slope of the tangent at P_1 (derivative of the function at P_1) is negative.

3. Take another two points say P_2 and P_3 on the same curve, and make tangents, using the same wire, at P_2 and P_3 making angles α_2 and α_3, respectively with the positive direction of x-axis.

4. Here again α_2 and α_3 are obtuse angles and therefore slopes of the tangents $\tan\alpha_2$ and $\tan\alpha_3$ are both negative, i.e., derivatives of the function at P_2 and P_3 are negative.

5. The function given by the curve (on the left) is a decreasing function.

6. On the curve (on the right), take three point Q_1, Q_2, Q_3, and using the other straight wires, form tangents at each of these points making angles β_1, β_2, β_3, respectively with the positive direction of x-axis, as shown in the figure. β_1, β_2, β_3 are all acute angles.

   So, the derivatives of the function at these points are positive. Thus, the function given by this curve (on the right) is an increasing function.

OBSERVATION

1. α_1 = _______ , > 90° α_2 = _______ > _______, α_3 = _______> _______, $\tan\alpha_1$ = _______, (negative) $\tan\alpha_2$ = _______, (_______), $\tan\alpha_3$ = _______, (_______). Thus the function is _______.

2. β_1 = _______< 90°, β_2 = _______, < _______, β_3 = _______ , < _______

   $\tan\beta_1$ = _______ , (positive), $\tan\beta_2$ = _______, (_______), $\tan\beta_3$ = _______(_______). Thus, the function is _______.

APPLICATION

This activity may be useful in explaining the concepts of decreasing and increasing functions.

Activity 14

Objective

To understand the concepts of local maxima, local minima and point of inflection.

Material Required

A piece of plywood, wires, adhesive, white paper.

Method of Construction

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wires each of length 40 cm and fix them on the paper on plywood in the form of *x*-axis and *y*-axis.
3. Take another wire of suitable length and bend it in the shape of curve. Fix this curved wire on the white paper pasted on plywood, as shown in Fig. 14.

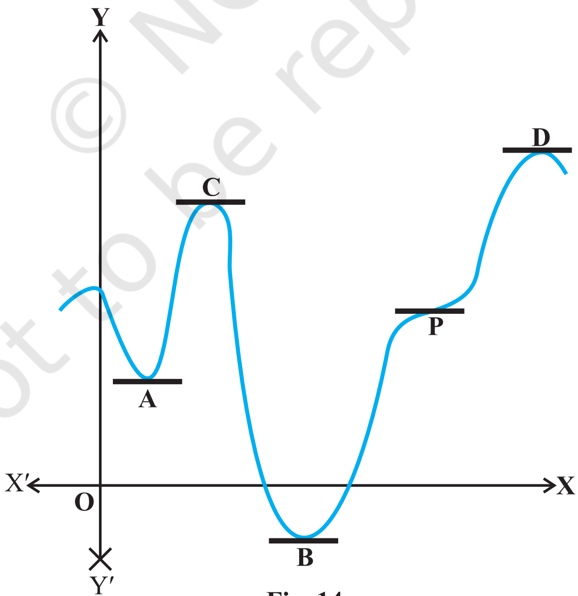


Fig. 14

4. Take five more wires each of length say 2 cm and fix them at the points A, C, B, P and D as shown in figure.

DEMONSTRATION

1. In the figure, wires at the points A, B, C and D represent tangents to the curve and are parallel to the axis. The slopes of tangents at these points are zero, i.e., the value of the first derivative at these points is zero. The tangent at P intersects the curve.
2. At the points A and B, sign of the first derivative changes from negative to positive. So, they are the points of local minima.
3. At the point C and D, sign of the first derivative changes from positive to negative. So, they are the points of local maxima.
4. At the point P, sign of first derivative does not change. So, it is a point of inflection.

OBSERVATION

1. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate left of A is _______.
2. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate right of A is_______.
3. Sign of the first derivative at a point on the curve to immediate left of B is _______.
4. Sign of the first derivative at a point on the curve to immediate right of B is _______.
5. Sign of the first derivative at a point on the curve to immediate left of C is _______.
6. Sign of the first derivative at a point on the curve to immediate right of C is _______.
7. Sign of the first derivative at a point on the curve to immediate left of D is _______.

8. Sign of the first derivative at a point on the curve to immediate right of D is _______.

9. Sign of the first derivative at a point immediate left of P is _______ and immediate right of P is_______.

10. A and B are points of local _______.

11. C and D are points of local _______.

12. P is a point of _______.

APPLICATION

1. This activity may help in explaining the concepts of points of local maxima, local minima and inflection.

2. The concepts of maxima/minima are useful in problems of daily life such as making of packages of maximum capacity at minimum cost.

Activity 15

OBJECTIVE

To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph.

MATERIAL REQUIRED

Drawing board, white chart paper, adhesive, geometry box, pencil and eraser, sketch pens, ruler, calculator.

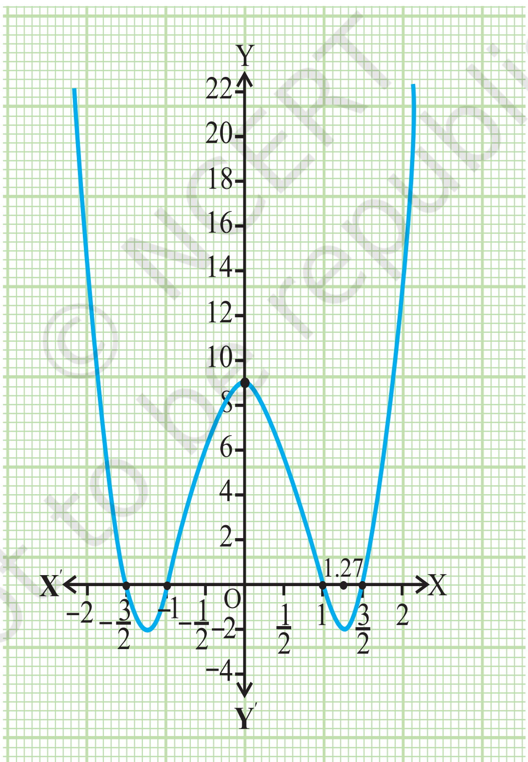


Fig 15

Method of Construction

1. Fix a white chart paper of convenient size on a drawing board using adhesive.
2. Draw two perpendicular lines on the squared paper as the two rectangular axes.
3. Graduate the two axes as shown in Fig.15.
4. Let the given function be $f(x) = (4x^2 - 9)(x^2 - 1)$ in the interval [–2, 2].
5. Taking different values of x in [–2, 2], find the values of $f(x)$ and plot the ordered pairs $(x, f(x))$.
6. Obtain the graph of the function by joining the plotted points by a free hand curve as shown in the figure.

Demonstration

1. Some ordered pairs satisfying $f(x)$ are as follows:

x	0	± 0.5	± 1.0	1.25	1.27	± 1.5	± 2
$f(x)$	9	6	0	– 1.55	–1.56	0	21

2. Plotting these points on the chart paper and joining the points by a free hand curve, the curve obtained is shown in the figure.

Observation

1. The absolute maximum value of $f(x)$ is ________ at x = ________.
2. Absolute minimum value of $f(x)$ is ________ at x = ________.

Application

The activity is useful in explaining the concepts of absolute maximum / minimum value of a function graphically.

NOTE

Consider $f(x) = (4x^2 - 9)(x^2 - 1)$

$f(x) = 0$ gives the values of x as $\pm\frac{3}{2}$ and ± 1. Both these values of x lie in the given closed interval [–2, 2].

$f'(x) = (4x^2 - 9)\, 2x + 8x\,(x^2 - 1) = 16x^3 - 26x = 2x\,(8x^2 - 13)$

$f'(x) = 0$ gives $x = 0,\ x = \pm\sqrt{\frac{13}{8}} = \pm 1.27$. These two values of x lie in [–2, 2].

The function has local maxima/minima at $x = 0$ and $x = \pm\ 1.27$, respectively.

Activity 16

Objective

To construct an open box of maximum volume from a given rectangular sheet by cutting equal squares from each corner.

Material Required

Chart papers, scissors, cellotape, calculator.

Method of Construction

1. Take a rectangular chart paper of size 20 cm × 10 cm and name it as ABCD.
2. Cut four equal squares each of side x cm from each corner A, B, C and D.
3. Repeat the process by taking the same size of chart papers and different values of x.
4. Make an open box by folding its flaps using cellotape/adhesive.

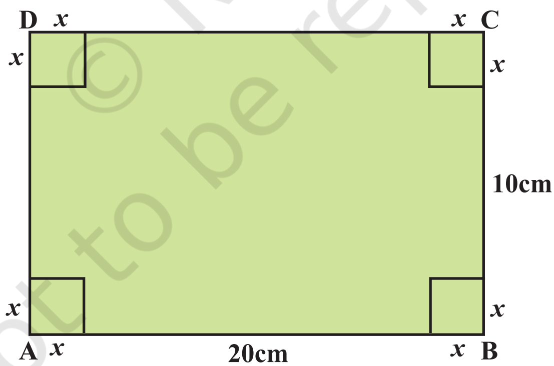


Fig. 16

Demonstration

1. When $x = 1$, Volume of the box = 144 cm^3
2. When $x = 1.5$, Volume of the box = 178.5 cm^3

3. When $x = 1.8$, Volume of the box = 188.9 cm^3.

4. When $x = 2$, Volume of the box = 192 cm^3.

5. When $x = 2.1$, Volume of the box = 192.4 cm^3.

6. When $x = 2.2$, Volume of the box = 192.2 cm^3.

7. When $x = 2.5$, Volume of the box = 187.5 cm^3.

8. When $x = 3$, Volume of the box = 168 cm^3.

Clearly, volume of the box is maximum when $x = 2.1$.

Observation

1. V_1 = Volume of the open box (when $x = 1.6$) = ...
2. V_2 = Volume of the open box (when $x = 1.9$) = ...
3. V = Volume of the open box (when $x = 2.1$) = ...
4. V_3 = Volume of the open box (when $x = 2.2$) = ...
5. V_4 = Volume of the open box (when $x = 2.4$) = ...
6. V_5 = Volume of the open box (when $x = 3.2$) = ...
7. Volume V_1 is ____________ than volume V.
8. Volume V_2 is ____________ than volume V.
9. Volume V_3 is ____________ than volume V.
10. Volume V_4 is ____________ than volume V.
11. Volume V_5 is ____________ than volume V.

So, Volume of the open box is maximum when x = ________.

Application

This activity is useful in explaining the concepts of maxima/minima of functions. It is also useful in making packages of maximum volume with minimum cost.

NOTE

Let V denote the volume of the box.

Now V = (20 – 2x) (10 – 2x) x

or V = 200x – 60x^2 + 4x^3

$\frac{dV}{dx} = 200 - 120x + 12x^2$. For maxima or minima, we have,

$\frac{dV}{dx} = 0$, i.e., $3x^2 - 30x + 50 = 0$

i.e., $x = \frac{30 \pm \sqrt{900 - 600}}{6} = 7.9$ or 2.1

Reject $x = 7.9$.

$\frac{d^2V}{dx^2} = -120 + 24x$

When $x = 2.1$, $\frac{d^2V}{dx^2}$ is negative.

Hence, V should be maximum at $x = 2.1$.

Activity 17

OBJECTIVE

To find the time when the area of a rectangle of given dimensions become maximum, if the length is decreasing and the breadth is increasing at given rates.

MATERIAL REQUIRED

Chart paper, paper cutter, scale, pencil, eraser, cardboard.

METHOD OF CONSTRUCTION

1. Take a rectangle R_1 of dimensions 16 cm × 8 cm.
2. Let the length of the rectangle is decreasing at the rate of 1cm/second and the breadth is increasing at the rate of 2 cm/second.
3. Cut other rectangle R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, etc. of dimensions 15 cm × 10 cm, 14 cm × 12 cm, 13 cm × 14 cm, 12 cm × 16 cm, 11 cm × 18 cm, 10 cm × 20 cm, 9 cm × 22 cm, 8 cm × 24 cm (see Fig.17).
4. Paste these rectangles on card board.

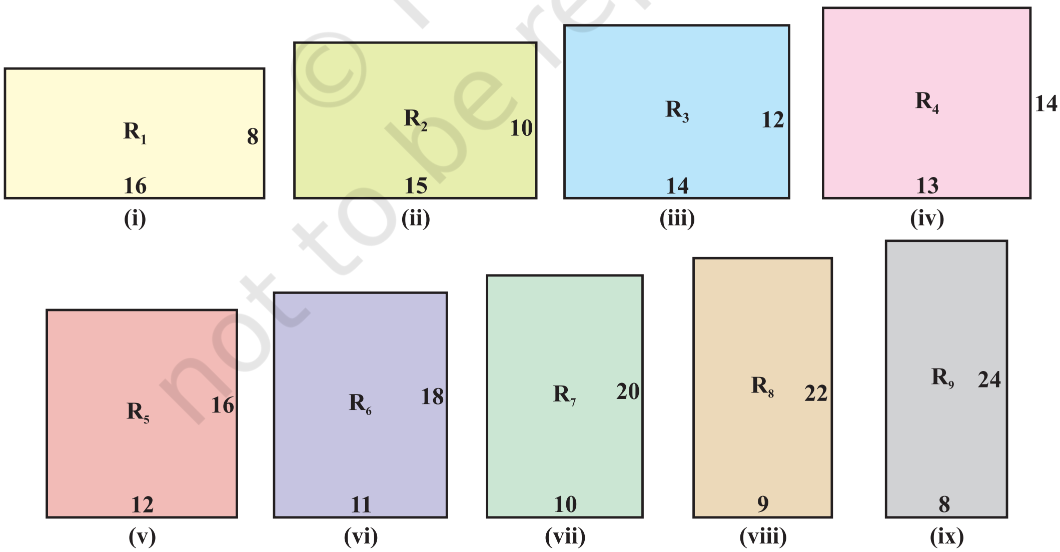


Fig. 17

DEMONSTRATION

1. Length of the rectangle is decreasing at the rate of 1cm/s and the breadth is increasing at the rate of 2cm/s.
2. (i) Area of the given rectangle $R_1 = 16 \times 8 = 128$ cm^2.

   (ii) Area of rectangle $R_2 = 15 \times 10 = 150$ cm^2 (after 1 sec).

   (iii) Area of rectangle $R_3 = 168$ cm^2 (after 2 sec).

   (iv) Area of rectangle $R_4 = 182$ cm^2 (after 3 sec).

   (v) Area of rectangle $R_5 = 192$ cm^2 (after 4 sec).

   (vi) Area of rectangle $R_6 = 198$ cm^2 (after 5 sec).

   (vii) Area of rectangle $R_7 = 200$ cm^2 (after 6 sec).

   (viii) Area of rectangle $R_8 = 198$ cm^2 (after 7 sec) and so on.

Thus the area of the rectangle is maximum after 6 sec.

OBSERVATION

1. Area of the rectangle R_2 (after 1 sec) = __________.
2. Area of the rectangle R_4 (after 3 sec) = __________.
3. Area of the rectangle R_6 (after 5 sec) = __________.
4. Area of the rectangle R_7 (after 6 sec) = __________.
5. Area of the rectangle R_8 (after 7 sec) = __________.
6. Area of the rectangle R_9 (after 8 sec) = __________.
7. Rectangle of Maximum area (after ..... seconds) = ________.
8. Area of the rectangle is maximum after _________ sec.
9. Maximum area of the rectangle is _________.

APPLICATION

This activity can be used in explaining the concept of rate of change and optimisation of a function.

The function has local maxima/minima at $x = 0$ and $x = \pm 1.27$, respectively.

NOTE

Let the length and breadth of rectangle be a and b.

The length of rectangle after t seconds = $a - t$.

The breadth of rectangle after t seconds = $b + 2t$.

Area of the rectangle (after t sec) = $A(t) = (a - t)(b + 2t) = ab - bt + 2at - 2t^2$

$A'(t) = -b + 2a - 4t$

For maxima or minima, $A'(t) = 0$.

$$A'(t)=0 \quad t \quad \frac{2a-b}{4}$$

$$A''(t)=-4$$

$$A'' \quad \frac{2a-b}{4} \quad =-4$$, which is negative

Thus, $A(t)$ is maximum at $t=\frac{2a-b}{4}$ seconds.

Here, a = 16 cm, b = 8 cm.

Thus, $t=\frac{32-8}{4}=\frac{24}{4}=6$ seconds

Hence, after 6 second, the area will become maximum.

Activity 18

OBJECTIVE

To verify that amongst all the rectangles of the same perimeter, the square has the maximum area.

MATERIAL REQUIRED

Chart paper, paper cutter, scale, pencil, eraser cardboard, glue.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Make rectangles each of perimeter say 48 cm on a chart paper. Rectangles of different dimensions are as follows:

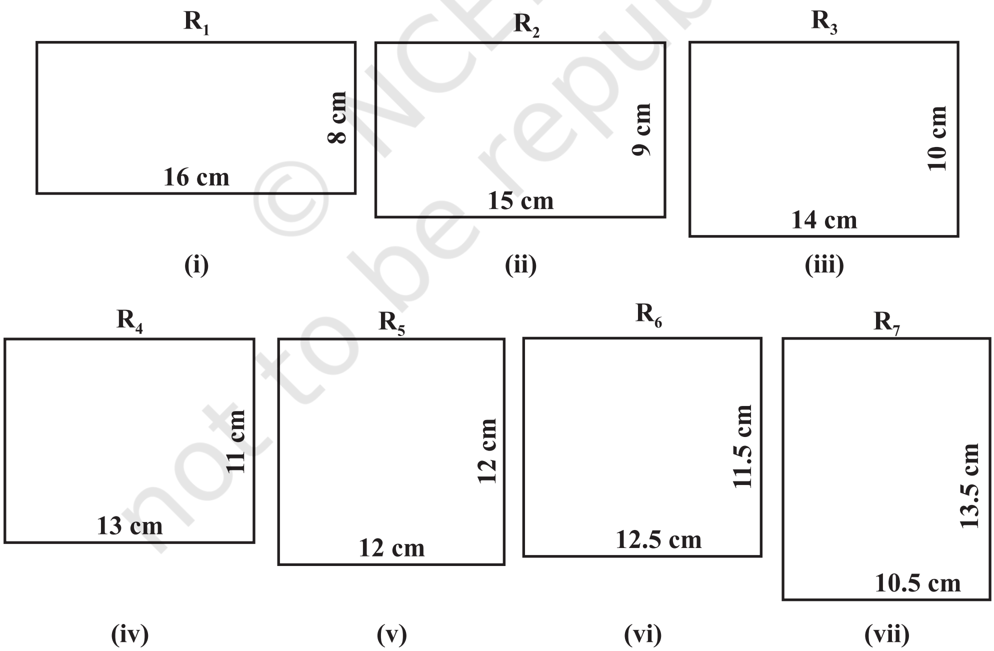


Fig. 18

R_1 : 16 cm × 8 cm, R_2 : 15 cm × 9 cm

R_3 : 14 cm × 10 cm, R_4 : 13 cm × 11 cm

R_5 : 12 cm × 12 cm, R_6 : 12.5 cm × 11.5 cm

R_7 : 10.5 cm × 13.5 cm

3. Cut out these rectangles and paste them on the white paper on the cardboard (see Fig. 18 (i) to (vii)).
4. Repeat step 2 for more rectangles of different dimensions each having perimeter 48 cm.
5. Paste these rectangles on cardboard.

DEMONSTRATION

1. Area of rectangle of R_1 = 16 cm × 8 cm = 128 cm^2

   Area of rectangle R_2 = 15 cm × 9 cm = 135 cm^2

   Area of R_3 = 140 cm^2

   Area of R_4 = 143 cm^2

   Area of R_5 = 144 cm^2

   Area of R_6 = 143.75 cm^2

   Area of R_7 = 141.75 cm^2
2. Perimeter of each rectangle is same but their area are different. Area of rectangle R_5 is the maximum. It is a square of side 12 cm. This can be verified using theoretical description given in the note.

OBSERVATION

1. Perimeter of each rectangle R_1, R_2, R_3, R_4, R_4, R_6, R_7 is _________.
2. Area of the rectangle R_3 ________ than the area of rectangle R_5.

3. Area of the rectangle R_6 _______ than the area of rectangle R_5.
4. The rectangle R_5 has the diamensions ______ × ______ and hence it is a _________.
5. Of all the rectangles with same perimeter, the ________ has the maximum area.

APPLICATION

This activity is useful in explaining the idea of Maximum of a function. The result is also useful in preparing economical packages.

NOTE

Let the length and breadth of rectangle be x and y.

The perimeter of the rectangle P = 48 cm.

$2(x + y) = 48$

or $x + y = 24$ or $y = 24 - x$

Let $A(x)$ be the area of rectangle, then

$A(x) = xy$

$= x(24 - x)$

$= 24x - x^2$

$A'(x) = 24 - 2x$

$A'(x) = \Rightarrow 24 - 2x = 0 \Rightarrow x = 12$

$A''(x) = -2$

$A''(12) = -2$, which is negative

Therefore, area is maximum when $x = 12$

$y = x = 24 - 12 = 12$

So, $x = y = 12$

Hence, amongst all rectangles, the square has the maximum area.

Activity 19

Objective

To evaluate the definite integral $\int_a^b \sqrt{(1-x^2)}\, dx$ as the limit of a sum and verify it by actual integration.

Material Required

Cardboard, white paper, scale, pencil, graph paper

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw two perpendicular lines to represent coordinate axes XOX′ and YOY′.
3. Draw a quadrant of a circle with O as centre and radius 1 unit (10 cm) as shown in Fig.19.

The curve in the 1st quadrant represents the graph of the function $\sqrt{1-x^2}$ in the interval [0, 1].

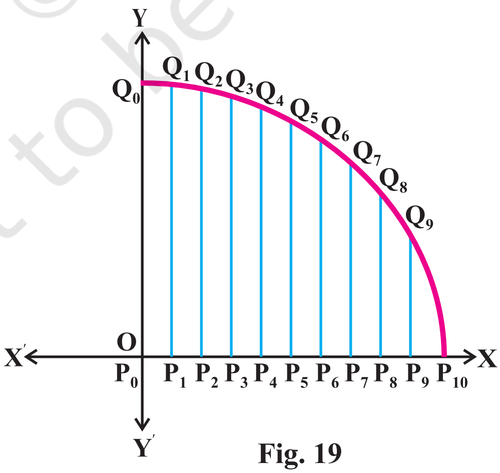


Fig. 19

DEMONSTRATION

1. Let origin O be denoted by P_0 and the points where the curve meets the x-axis and y-axis be denoted by P_{10} and Q, respectively.
2. Divide P_0P_{10} into 10 equal parts with points of division as, P_1, P_2, P_3, ..., P_9.
3. From each of the points, P_i , $i = 1, 2, ..., 9$ draw perpendiculars on the x-axis to meet the curve at the points, Q_1, Q_2, Q_3 ,..., Q_9. Measure the lengths of P_0Q_0, $P_1 Q_1$, ..., P_9Q_9 and call them as y_0, y_1 , ..., y_9 whereas width of each part, P_0P_1, P_1P_2, ..., is 0.1 units.
4. $y_0 = P_0Q_0 = 1$ units

   $y_1 = P_1Q_1 = 0.99$ units

   $y_2 = P_2Q_2 = 0.97$ units

   $y_3 = P_3Q_3 = 0.95$ units

   $y_4 = P_4Q_4 = 0.92$ units

   $y_5 = P_5Q_5 = 0.87$ units

   $y_6 = P_6Q_6 = 0.8$ units

   $y_7 = P_7Q_7 = 0.71$ units

   $y_8 = P_8Q_8 = 0.6$ units

   $y_9 = P_9Q_9 = 0.43$ units

   $y_{10} = P_{10}Q_{10}$ = which is very small near to 0.
5. Area of the quadrant of the circle (area bounded by the curve and the two axis) = sum of the areas of trapeziums.

$$= \frac{1}{2} \times 0.1 \begin{bmatrix} (1+0.99)+(0.99+0.97)+(0.97+0.95)+(0.95+0.92) \\ +(0.92+0.87)+(0.87+0.8)+(0.8+0.71)+(0.71+0.6) \\ +(0.6+0.43)+(0.43) \end{bmatrix}$$

$= 0.1 [0.5 + 0.99 + 0.97 + 0.95 + 0.92 + 0.87 + 0.80 + 0.71 + 0.60 + 0.43]$

$= 0.1 \times 7.74 = 0.774$ sq. units.(approx.)

6. Definite integral $= \int_0^1 \sqrt{1-x^2}\, dx$

$$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1} x \right]_0^1 = \frac{1}{2} \times \frac{\pi}{2} = \frac{3.14}{4} = 0.785 \text{ sq.units}$$

Thus, the area of the quadrant as a limit of a sum is nearly the same as area obtained by actual integration.

Observation

1. Function representing the arc of the quadrant of the circle is y = ______.
2. Area of the quadrant of a circle with radius 1 unit $= \int_0^1 \sqrt{1-x^2}\, dx$ = ________.

   sq. units
3. Area of the quadrant as a limit of a sum = _______ sq. units.
4. The two areas are nearly _________.

Application

This activity can be used to demonstrate the concept of area bounded by a curve. This activity can also be applied to find the approximate value of π.

Note

Demonstrate the same activity by drawing the circle $x^2 + y^2 = 9$ and find the area between $x = 1$ and $x = 2$.

Activity 20

Objective

To verify geometrically that

$\vec{c}\times(\vec{a}+\vec{b})=\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$

Material Required

Geometry box, cardboard, white paper, cutter, sketch pen, cellotape.

Method of Construction

1. Fix a white paper on the cardboard.
2. Draw a line segment OA (= 6 cm, say) and let it represent $\vec{c}$.
3. Draw another line segment OB (= 4 cm, say) at an angle (say 60°) with OA.

   Let $\overrightarrow{OB} = \vec{a}$

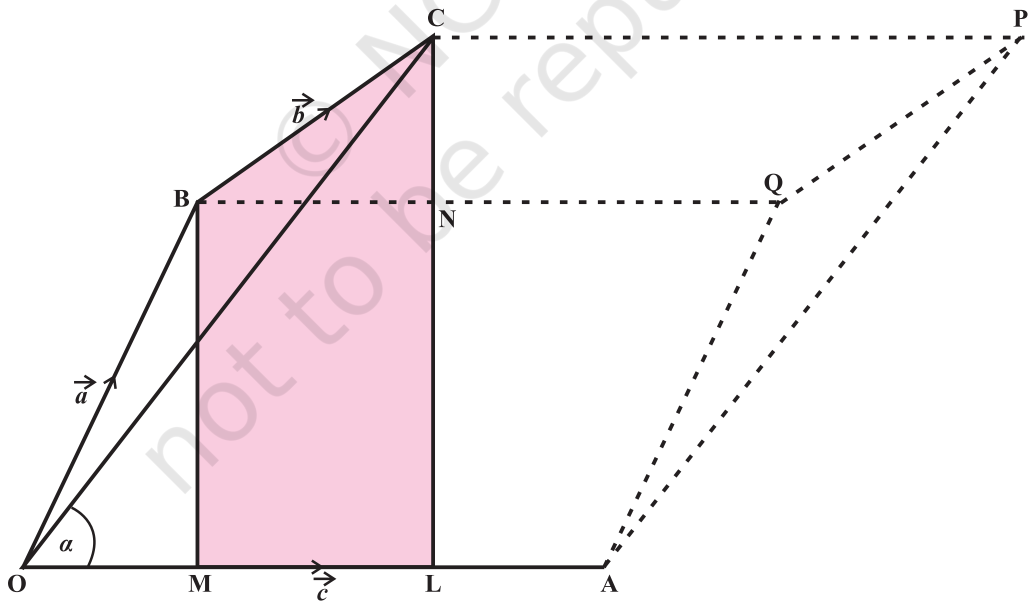


Fig. 20

4. Draw BC (= 3 cm, say) making an angle (say 30°) with $\overrightarrow{OA}$. Let $\overrightarrow{BC} = \vec{b}$

5. Draw perpendiculars BM, CL and BN.

6. Complete parallelograms OAPC, OAQB and BQPC.

Demonstration

1. $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \vec{a} + \vec{b}$, and let $\angle COA = \alpha$.

2. $\left|\vec{c} \times \left(\vec{a} + \vec{b}\right)\right| = \left|\vec{c}\right| \left|\vec{a} + \vec{b}\right|$ sin α = area of parallelogram OAPC.

3. $\left|\vec{c} \times \vec{a}\right|$ = area of parallelogram OAQB.

4. $\left|\vec{c} \times \vec{b}\right|$ = area of parallelogram BQPC.

5. Area of parallelogram OAPC = (OA) (CL)

= (OA) (LN + NC) = (OA) (BM + NC)

= (OA) (BM) + (OA) (NC)

= Area of parallelogram OAQB + Area of parallelogram BQPC

$= \left|\vec{c} + \vec{a}\right| + \left|\vec{c} \times \vec{b}\right|$

So, $\left|\vec{c} \times (\vec{a} + \vec{b})\right| = \left|\vec{c} \times \vec{b}\right| + \left|\vec{c} \times \vec{b}\right|$

Direction of each of these vectors $\vec{c} \times (\vec{a} + \vec{b})$, $\vec{c} \times \vec{a}$ and $\vec{c} \times \vec{b}$ is perpendicular to the same plane.

So, $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$.

OBSERVATION

$|\vec{c}| = |\overrightarrow{OA}| = OA =$ _______

$|\vec{a}+\vec{b}| = |\overrightarrow{OC}| = OC =$ ______

CL = ______

$|\vec{c}\times(\vec{a}+\vec{b})|$ = Area of parallelogram OAPC

= (OA) (CL) = _____________ sq. units (i)

$|\vec{c}\times\vec{a}|$ = Area of parallelogram OAQB

= (OA) (BM) = _____ × _____ = ______ (ii)

$|\vec{c}\times\vec{b}|$ = Area of parallelogram BQPC

= (OA) (CN) = _____ × _____ = ______ (iii)

From (i), (ii) and (iii),

Area of parallelogram OAPC = Area of parallelgram OAQB + Area of Parallelgram ________.

Thus $|\vec{c}\times(\vec{a}+\vec{b})| = |\vec{c}\times\vec{a}| + |\vec{c}\times\vec{b}|$

$\vec{c}\times\vec{a}$, $\vec{c}\times\vec{b}$ and $\vec{c}\times(\vec{a}+\vec{b})$ are all in the direction of _______ to the plane of paper.

Therefore $\vec{c}\times(\vec{a}+\vec{b}) = \vec{c}\times\vec{a} +$ ________.

APPLICATION

Through the activity, distributive property of vector multiplication over addition can be explained.

NOTE

This activity can also be performed by taking rectangles instead of parallelograms.

Activity 21

Objective

To verify that angle in a semi-circle is a right angle, using vector method.

Material Required

Cardboard, white paper, adhesive, pens, geometry box, eraser, wires, paper arrow heads.

Method of Construction

1. Take a thick cardboard of size 30 cm × 30 cm.
2. On the cardboard, paste a white paper of the same size using an adhesive.
3. On this paper draw a circle, with centre O and radius 10 cm.

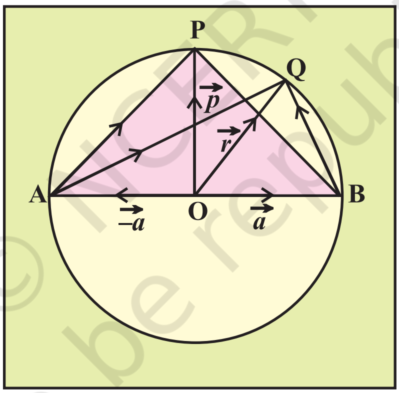


Fig. 21

4. Fix nails at the points O, A, B, P and Q. Join OP, OA, OB, AP, AQ, BQ, OQ and BP using wires.
5. Put arrows on OA, OB, OP, AP, BP, OQ, AQ and BQ to show them as vectors, using paper arrow heads, as shown in the figure.

Demonstration

1. Using a protractor, measure the angle between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$, i.e., $\angle$ APB = 90°.

2. Similarly, the angle between the vectors $\overrightarrow{AQ}$ and $\overrightarrow{BQ}$, i.e., $\angle AQB = 90°$.

3. Repeat the above process by taking some more points R, S, T, ... on the semi-circles, forming vectors AR, BR; AS, BS; AT, BT; ..., etc., i.e., angle formed between two vectors in a semi-circle is a right angle.

Observation

By actual measurement.

$\left|\overrightarrow{OP}\right| = \left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right| = \left|\overrightarrow{OQ}\right| = r = a = p =$ _______ ,

$\left|\overrightarrow{AP}\right|$ = _______ , $\left|\overrightarrow{BP}\right|$ = _______, $\left|\overrightarrow{AB}\right|$ = ______

$\left|\overrightarrow{AQ}\right|$ = _______ , $\left|\overrightarrow{BQ}\right|$ = _______

$\left|\overrightarrow{AP}\right|^2 + \left|\overrightarrow{BP}\right|^2$ = ________, $\left|\overrightarrow{AQ}\right|^2 + \left|\overrightarrow{BQ}\right|^2$ = ________

So, $\angle APB$ = ________ and $\overrightarrow{AP}.\overrightarrow{BP}$ ________ $\angle AQB$ = ________ and $\overrightarrow{AQ}.\overrightarrow{BP}$ = ________

Similarly, for points R, S, T, ________

$\angle ARB$ = ________, $\angle ASB$ = ________, $\angle ATB$ = ________, ________

i.e., angle in a semi-circle is a right angle.

Application

This activity can be used to explain the concepts of

(i) opposite vectors

(ii) vectors of equal magnitude

(iii) perpendicular vectors

(iv) Dot product of two vectors.

NOTE

Let OA = OB = *a* = OP = *p*

$\overrightarrow{OA} = -\vec{a}$, $\overrightarrow{OB} = \vec{a}$, $\overrightarrow{OP} = \vec{p}$

$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP} = \vec{a} + \vec{p}$., $\overrightarrow{BP} = \vec{p} - \vec{a}$.

$$\overrightarrow{AP}.\overrightarrow{BP} = (\vec{p}+\vec{a}).(\vec{p}-\vec{a}) = |\vec{p}|^2 - |\vec{a}|^2 = 0$$

$$\left(\text{since} |\vec{p}|^2 = |\vec{a}|\right)$$

So, the angle APB between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ is a right angle.

Similarly, $\overrightarrow{AQ}.\overrightarrow{BQ} = 0$, so, $\angle AQB = 90°$ and so on.

Activity 22

Objective

To locate the points to given coordinates in space, measure the distance between two points in space and then to verify the distance using distance formula.

Material Required

Drawing board, geometry box, squared paper, nails of different lengths, paper arrows.

Method of Construction

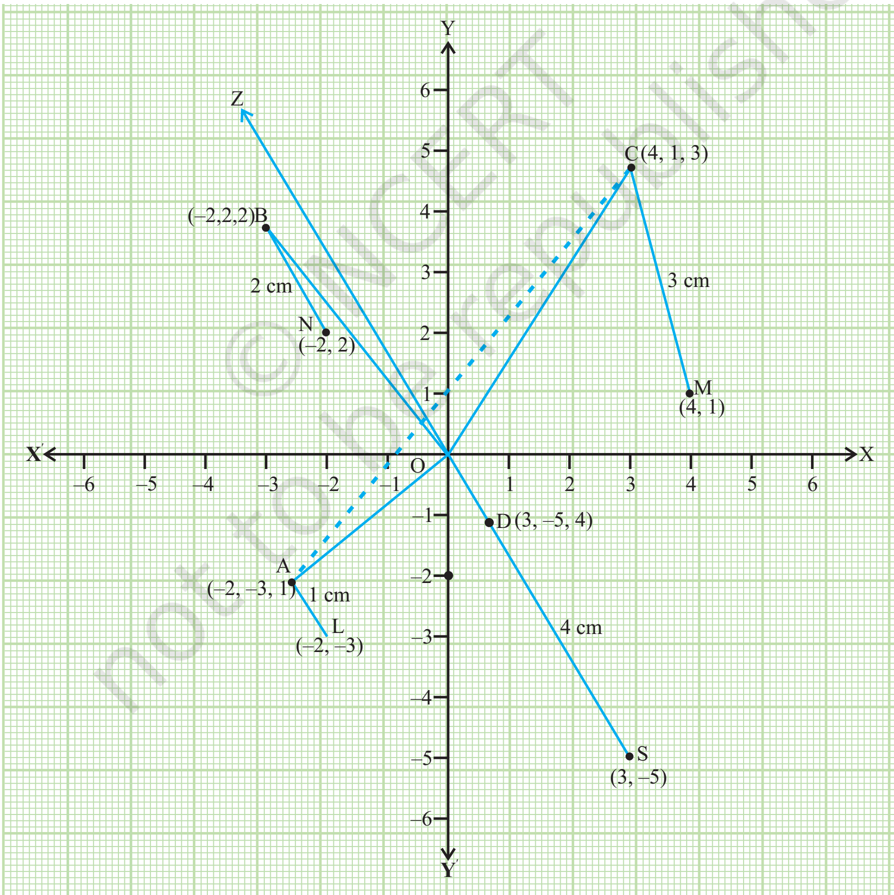


Fig 22

1. Take a drawing board and paste a squared paper on it.
2. Draw two lines X′OX and Y′OY to represent *x*-axis, *y*-axis respectively (see Fig. 22) and take 1 unit = 1 cm.
3. Fix a wire through O, in the vertical direction, representing the *z*-axis.
4. Fix nails of length 1 cm, 2 cm, 3 cm, 4 cm, etc. at different points on the squared paper (say at L (–2, –3), N (–2, 2), M (4, 1), S (3, –5)), etc.

Now the upper tips of these nails represent the points (say A, B, C, D) in the space.

Demonstration

1. Coordinates of the point A = (–2, –3, 1).
2. Coordinates of the point B = (–2, 2, 2).
3. Similarly find the coordinates of the point C and D.
4. By actual measurement (using a scale) distance AB = 5.1 cm.
5. By distance formula, $AB=\sqrt{(-2+2)^2+(-3-2)^2+(1-2)^2} = \sqrt{26} = 5.099$.

Thus, the distance AB, obtained by actual measurement is approximately same as the distance obtained by using the distance formula.

Same can be verified for other pairs of points A, C; B, C; A, D; C, D; B, D.

Observation

Coordinates of the point C = ________.

Coordinates of the point D = ________.

On actual measurement :

AC = ________, BC = __________.

AD = ________, CD = __________, BD = ___________.

Using distance formula; AC = ________, BC = ________, AD = ________

CD = ________, BD = ________.

Thus, the distance between two points in space obtained on actual measurement and by using distance formula is approximately the same.

APPLICATION

1. This activity is useful in visualising the position of different points in space (coordinates of points).
2. The concept of position vectors can also be explained through this activity.

Activity 23

Objective

To demonstrate the equation of a plane in normal form.

Material Required

Two pieces of plywood of size 10 cm × 12 cm, a thin wooden rod with nuts and bolts fixed on both sides, 3 pieces of wires, pen/pencil.

Method of Construction

1. Fix the wooden rod in between two wooden pieces with nuts and bolts so that the rod is perpendicular to the two wooden pieces. So, it represents the normal to the plane.

2. Take three wires and fix, them as shown in Fig. 23 so that $\overrightarrow{OP}$ represents the vector $\vec{a}$ and $\overrightarrow{OA}$ represents $\vec{r}$. Then the wire $\overrightarrow{PA}$ represents the vector $\vec{r}-\vec{a}$.

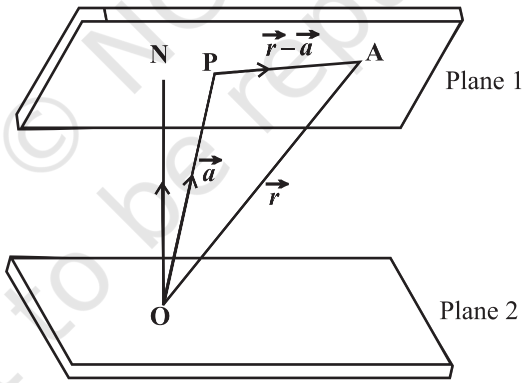


Fig. 23

Demonstration

1. The wire PA, i.e., the vector $(\vec{r}-\vec{a})$ lies on plane 1. On representing $\vec{n}$ as normal to plane 1, $\hat{n}$ is perpendicular to $(\vec{r}-\vec{a})$, normal to the plane.

2. Hence $(\vec{r}-\vec{a})\cdot\vec{n}=0$ which gives the equation of plane in the normal form.

OBSERVATION

1. $\vec{a}$ is the position vector of __________, $\vec{r}$ is the position vector of __________ vector $\vec{n}$ is perpendicular to the vector ___________.

2. $(\vec{r}-\vec{a}).\ \hat{n}=0$, is the equation of the plane _________, in ________ form.

APPLICATION

This activity can also be utilised to show the position vector of a point in space (i.e., $\vec{a}$ as position vector of O, $\vec{r}$ the position vector of A).

Activity 24

Objective

To verify that the angle between two planes is the same as the angle between their normals.

Material Required

Plywood pieces, wires, hinges.

Method of Construction

1. Take two pieces of plywood 10 cm × 20 cm and join them with the help of hinges.
2. Fix two vertical wires on each plane to show normals to the planes.
3. Cut slots in the two planes to fix a third plywood piece showing third plane.

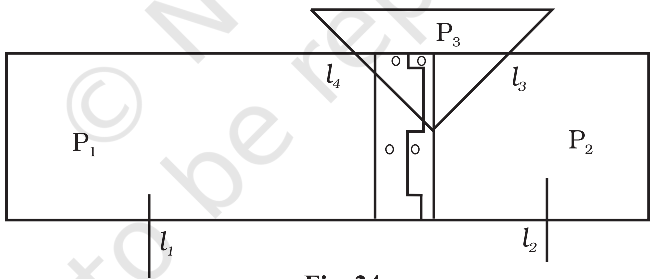


Fig. 24

Demonstration

1. P_1 represents the first plane.
2. P_2 represents the second plane.
3. Vertical wires l_1 and l_2 represents normals to the planes P_1, P_2, respectively.

4. l_3 and l_4 are the lines of intersections of the planes P_3, with P_1 and P_2, respectively.
5. Angle between lines l_3 and l_4 is the angle between the planes. It is same as the angle between their normals.

Observation

1. P_1 represents the _________.
2. P_2 represents the _________.
3. l_1 represents the _________.
4. l_2 represents the _________.
5. l_3 is the line of intersection _________.
6. l_4 is the line of intersection _________.
7. Angle between l_1 and l_2 is equal to _________.

Application

This model can also be used to find the angle between a line and a plane.

Activity 25

Objective

To find the distance of given point (in space) from a plane (passing through three non-collinear points) by actual measurement and also analytically.

Material Required

One cardboard of size 20 cm × 30 cm and another of size 10 cm × 15 cm., a thick sheet of paper of size 20 cm × 30 cm, nails of varying lengths with caps on one end, geometry box, wires.

Method of Construction

1. Draw two mutually perpendicular lines X′OX and Y′OY on a thick sheet of paper representing *x*-axis, and *y*-axis, respectively intersecting at O, and graduate them.

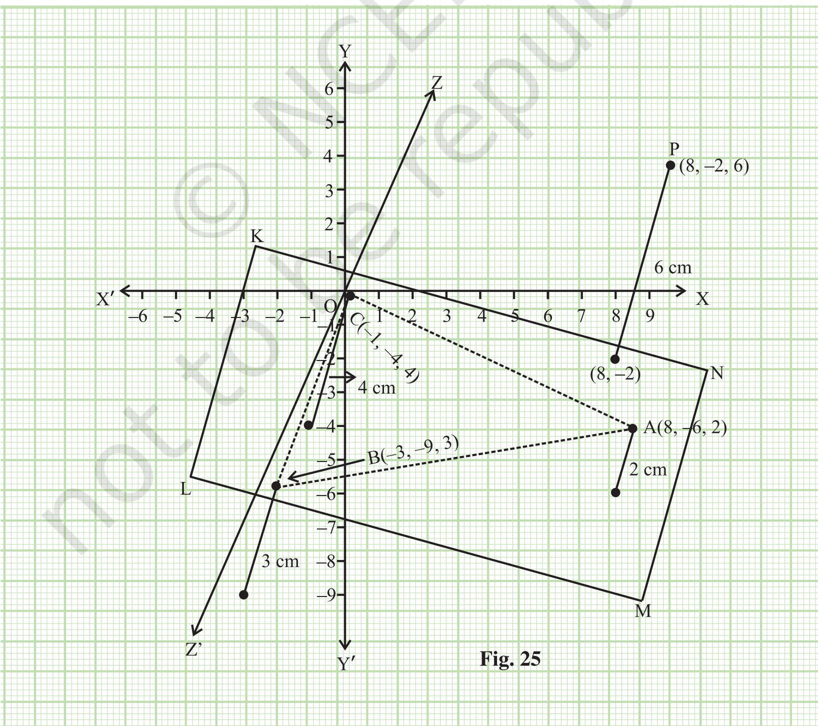


Fig. 25

2. Paste this sheet on the cardboard of size 20 cm × 30 cm. Through O, fix a wire vertically to represent z-axis (see Fig. 25).
3. Fix three nails of heights (say 2 cm, 3 cm and 4 cm) at three different points on this board (say at (8, –6), (–3, –9) and (–1, –4), respectevely) (Fig. 25).
4. The tips of these nails represent three points A, B and C in space.
5. Now rest a plane KLMN represented by another cardboard on the tips of these three nails so that the points A, B, C, lie on the plane.
6. Now fix a nail of length 6 cm at some point [say (8, –2)] on the cardboard.The tip of the nail will represent point P, from where the distance to the plane KLMN is to be found.

Demonstration

1. Coordinates of the points A, B and C are (8, –6, 2), (–3, –9, 3), (–1, – 4, 4), respectively.
2. Coordinates of point P are (8, –2, 6).
3. A set square is placed so that its one side-forming the right angle on the plane KLMN and the other side in the direction normal to the plane.
4. Place a metre scale along the side of the set square which is in the direction normal to the plane KLMN and slide both of them until the metre scale touches the point P.
5. The distance between the point P and the plane in the normal direction is measured using a metre scale.
6. Equation of the plane passing through the points A, B, C is

$$\begin{vmatrix} x-8 & y+6 & z-2 \\ -3-8 & -9+6 & 3-2 \\ -1-8 & -4+6 & 4-2 \end{vmatrix} = 0 \text{ which is of the form } ax + by + cz + d = 0.$$

7. This distance is also calculated by using the formula

$$d = \left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|.$$

8. The two distance so obtained are the same.

Observation

1. The coordinates of A (x_1, y_1, z_1) = __________.

   B (x_2, y_2, z_2) = __________.

   C (x_3, y_3, z_3) = __________.

   Coordinates of point P = __________.

   The distance(d) of P from the plane KLMN by actual measurement =__________.

2. Equation of plane through A, B, C using

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$ is __________.

   The distance of P from this plane (represented by above equation)

   using $d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ = _________.

   Thus distance of a point P from a plane by actual measurement

   = distance of P through analytical method = ___________.

APPLICATION

1. With this activity it can be explained that through

   (a) one point or through two points, infinite number of planes can pass,

   (b) three non-collinear points, a unique plane passes.

2. This activity can also be used in explainiing the concept of distance between two parallel planes.

Activity 26

Objective

To measure the shortest distance between two skew lines and verify it analytically.

Material Required

A piece of plywood of size 30 cm × 20 cm, a squared paper, three wooden blocks of size 2cm × 2 cm × 2 cm each and one wooden block of size 2 cm × 2 cm × 4 cm, wires of different lengths, set squares, adhesive, pen/pencil, etc.

Method of Construction

1. Paste a squared paper on a piece of plywood.
2. On the squared paper, draw two lines OA and OB to represent x-axis, and y-axis, respectively.
3. Name the three blocks of size 2 cm × 2 cm × 2 cm as I, II and III. Name the other wooden block of size 2 cm × 2 cm × 4 cm as IV.
4. Place blocks I, II, III such that their base centres are at the points (2, 2), (1, 6) and (7, 6), respectively, and block IV with its base centre at (6, 2). Other wooden block of size 2 cm × 2 cm × 4 cm as IV.
5. Place a wire joining the points P and Q, the centres of the bases of the blocks I and III and another wire joining the centres R and S of the tops of blocks II and IV as shown in Fig. 26.
6. These two wires represent two skew lines.
7. Take a wire and join it perpendicularly with the skew lines and measure the actual distance.

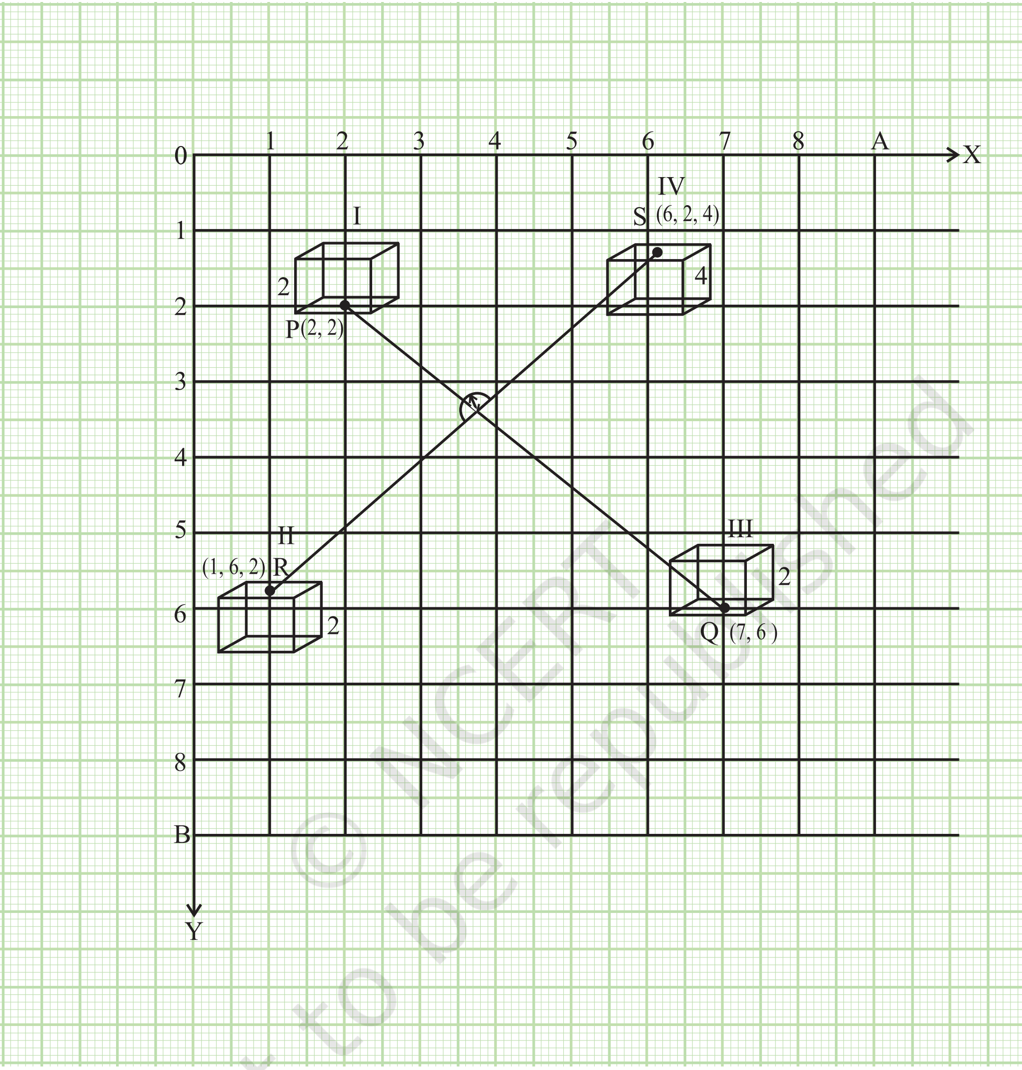


Fig. 26

Demonstration

1. A set-square is placed in such a way that its one perpendicular side is along the wire PQ.
2. Move the set-square along PQ till its other perpendicular side touches the other wire.

3. Measure the distance between the two lines in this position using set-square. This is the shortest distance between two skew lines.
4. Analytically, find the equation of line joining P (2, 2, 0) and Q (7, 6, 0) and other line joining R (1, 6, 2) and S (6, 2, 4) and find S.D. using $\frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}$. The distance obtained in two cases will be the same.

Observation

1. Coordinates of point P are ________.
2. Coordinates of point Q are ________.
3. Coordinates of point R are ________.
4. Coordinates of point S are ________.
5. Equation of line PQ is ________.
6. Equation of line RS is ________.

Shortest distance between PQ and RS analytically = ________.

Shortest distance by actual measurement = ________.

The results so obtained are ________.

Application

This activity can be used to explain the concept of skew lines and of shortest distance between two lines in space.

Activity 27

Objective

To explain the computation of conditional probability of a given event A, when event B has already occurred, through an example of throwing a pair of dice.

Material Required

A piece of plywood, white paper pen/pencil, scale, a pair of dice.

Method of Construction

1. Paste a white paper on a piece of plywood of a convenient size.
2. Make a square and divide it into 36 unit squares of size 1cm each (see Fig. 27).
3. Write pair of numbers as shown in the figure.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Fig. 27

Demonstration

1. Fig. 27 gives all possible outcomes of the given experiment. Hence, it represents the sample space of the experiment.
2. Suppose we have to find the conditional probability of an event A if an event B has already occurred, where A is the event "a number 4 appears on both the dice" and B is the event "4 has appeared on at least one of the dice"i.e, we have to find P(A | B).
3. From Fig. 27 number of outcomes favourable to A = 1

   Number of outcomes favourable to B = 11

   Number of outcomes favourable to A ∩ B = 1.
4. (i) $P(B) = \frac{11}{36}$,

   (ii) $P(A \cap B) = \frac{1}{36}$

   (iii) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{11}$.

Note

1. You may repeat this activity by taking more events such as the probability of getting a sum 10 when a doublet has already occurred.

2. Conditional probability P (A | B) can also be found by first taking the sample space of event B out of the sample space of the experiment, and then finding the probability A from it.

Observation

1. Outcome(s) favourable to A : __________, n (A) = __________.
2. Outcomes favourable to B : __________, n (B) = __________.
3. Outcomes favourable to A ∩ B : __________, n (A ∩ B) = __________.
4. P (A ∩ B) = __________.
5. P (A | B) = __________ = __________.

Application

This activity is helpful in understanding the concept of conditional probability, which is further used in Bayes' theorem.