

13th KVS Maths Olympiad Contest – 2010

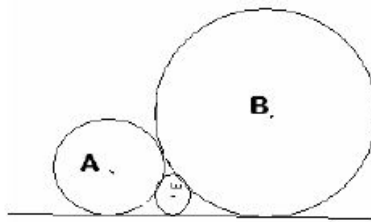
M.M. 100

Time : 3 hours

Note : (i) Please check that there are two printed pages and ten questions in all.

(ii) Attempt all questions. All questions carry equal marks

- 1) Let a, b, c are real numbers a not equal to zero such that a and $4a + 3b + 2c$ have the same sign. Show that the equation $ax^2 + bx + c = 0$ can not have both roots in the interval $(1, 2)$.
- 2) a) find all the integers which are equal to 11 times the sum of their digits.
b) Prove that $\sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}}$ is a rational number.
- 3) A circle centered at A with radius 1 unit and another circle centered at B with radius 4 units touch each other externally. A third circle is drawn to touch the first two circles and one of their common external tangents as shown in the figure. What is the radius of the third circle?



- 4) Given three non co-linear points A, H and G . Construct a triangle with A as vertex, H as orthocenter and G as centroid.
- 5) A triangle ABC , angle A is twice the angle B . Prove that $a^2 = b(b+c)$ where

a, b and c are the sides opposite to angles A, B and C respectively.

- 6) The equation $x^2+px+q = 0$, where p and q are integers, has rational roots.
Prove that the roots are integers.
- 7) Triangle ABC is right triangle with angle C is 90° . Measure of $\angle ABC = 60^\circ$, and $AB = 10$ units. Let P be a point chosen randomly inside triangle ABC. Extend BP to meet AC at D. What is the probability that $BD > 5\sqrt{2}$.
- 8) Prove that the sum of hypotenuse and the altitude of a right angled triangle dropped on the hypotenuse exceed the half perimeter of the triangle.
- 9) How many times is digit zero is written when listing all numbers from 1 to 3333?
- 10) Find out the remainder when $x+x^9+x^{25}+x^{49}+x^{81}$ is divided by x^3-x ?