KVS Junior Mathematics Olympiad (JMO)

SAMPLE PAPER – 4

M.M. 100 Time : 3 hours

Note : Attempt all questions.
All questions carry equal marks

Q1. Factorise : $(a+b+c)^3 - a^3 - b^3 - c^3$

Q2. Determine the kind of a triangle if it is known that that its medians are related by the equality

$m_a^2 + m_b^2 = 5m_c^2$, m_a : stands for median through angle A.

Q3. Transpose letter to digits :

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      A   B   C    &      A   D   G
+     D   E   F      +    B   E   H
   -------------       -------------
      G   H   I           C   F   I
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The digit value is same for same letter in both the problem.

Q4. Find seven different unit factors (volume of a,b,c,d,e,f and g) Whose sum is 1.

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}+\frac{1}{f}+\frac{1}{g}=1$$

Q5. Find the remainder when 7^{84} is divided by 342.

Q6. Two circles of radius r are externally tangent. They are also internally tangent to the sides of a right triangle of 6, 8 and 10, with the hypotenuse of the triangle being tangent to both circles. Find the value of radius r.

Q7. A journey of 192 KM from Mumbai to Pune takes 2 hours less by a superfast train than 2 hour's less by a super fast train than that by an ordinary passenger train. If the average speed of the lower train is 16 KM /h less than that of the faster train, find their average speeds.

Q8. Given a list of 1998 odd numbers, is it possible that the square of one of them is equal to sum of the squares of the other 1997 numbers ?

Q9. The height of a cone is 30 cm. A small cone is cut off at the top by a place parallel to the base. If its volume be 1/27 of the volume of the given cone, at what height above the base is the section made ?

Q10. Evaluate

$$\left[1+\frac{1}{2^2-1}\right]\left[1+\frac{1}{3^2-1}\right]\ \cdots\ \left[1+\frac{1}{99^2+1}\right]$$

....

SOLUTIONS AND HINT (SAMPLE PAPER – 4)

Q. 1. $(a+b+c)^3 - a^3 - b^3 - c^3$

$= (a+b)^3 + 3(a+b)\ .c.\ (a+b+c) + c^3 - a^3 - b^3 - c^3$

$= 3ab\,(a+b) + 3c\,(a+b)\ (a+b+c)$

$= 3(a+b)\ \{ab + c\,(a+b+c)\}$

$= 3(a+b)\ \left\{ab + ac + bc + c^2\right\}$

$= 3(a+b)\ \{b\,(b+c) + c\,(b+c)\}$

$= 3(a+b)(b+c)(a+c)$

$= 3(a+b)(b+c)(c+a)$

Q. 2 We Know

$$m_a = \frac{\sqrt{2a^2 + 2c^2 - a^2}}{2}$$

$$m_b = \frac{\sqrt{2a^2 + 2c^2 - b^2}}{2}$$

$$m_c = \frac{\sqrt{2a^2 + 2b^2 - c^2}}{2}$$

Given that, $m_a^2 + m_b^2 = 5m_c^2$

$$\Rightarrow \frac{2b^2 + 2c^2 - a^2}{4} + \frac{2a^2 + 2c^2 - b^2}{4} = 5\ \frac{\left(2a^2 + 2b^2 - c^2\right)}{4}$$

$\Rightarrow 9a^2 + 9b^2 = 9c^2$

$\Rightarrow a^2 + b^2 = c^2$ So, angle $\angle c = 90^0$, A right triangle

Q. 3 The solution only is noted below, Arrived by hit and trial.

$$\begin{array}{r} 157 \\ +\ 482 \\ \hline 639 \end{array} \quad \& \quad \begin{array}{r} 146 \\ +\ 583 \\ \hline 729 \end{array}$$

Second Solution

$$\begin{array}{r} 729 \\ +\ 135 \\ \hline 864 \end{array} \quad \& \quad \begin{array}{r} 718 \\ +\ 236 \\ \hline 954 \end{array}$$

Q. 4 Solution is

a	2	3	2	3	3	3	
b	6	5	4	4	5	4	
c	8	6	8	6	6	7	
d	12	8	16	9	9	8	
e	20	10	32	13	12	14	
f	24	20	34	18	18	24	
g	30	40	544	156	20	28	

Q.5 7^{84} by 342

We know , $7^3 = 7 \; x \; 7 \; x \; 7$

$= 343$

$\Rightarrow \; 7^3 \equiv 343 \pmod{342}$

$\Rightarrow \; 7^3 \equiv 1 \pmod{342}$

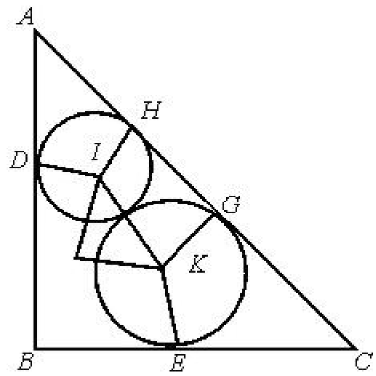
$\Rightarrow \left(7^3\right)^{28} \equiv (1)^{28} \pmod{342}$

$\Rightarrow 7^{84} \equiv (1)^{28} \pmod{342}$

$\Rightarrow 7^{84} \equiv 1 \pmod{342}$

$\therefore \; 7^{84}$ leaves a remainder of 1 on division by 342.

Q. 6



Let ABC in a 6, 8, 10 right triangle.

AB = 6 BC = 8 CA=10

x = AH = AD

y = CG = CE

So, JK = 8-r-y

IJ = 6-r-x, and

IK = 2r

= 10-x-y

Since Δ IJK is similar to Δ ABC

$$\frac{8-r-y}{2r} = \frac{8}{10} \Rightarrow 15r = 40 - 5y$$

$$\frac{6-r-x}{2r}=\frac{6}{10}\Rightarrow 11r=30-5x$$

Further, 2r = 10 – x – y

$$\Rightarrow 2r=10+\frac{26x-70}{5}$$

$$\Rightarrow \quad r=\frac{5}{4}$$

Q. 7. Let, speed of slow train = x km/h and

Speed of fast tran = y KM/h

So, from question

$$\frac{192}{x}-\frac{192}{y}=2 \qquad \text{.....I}$$

further y - x = 16 II

From (i) $\frac{y-x}{xy}=\frac{2}{192}$

$$\Rightarrow xy = 8 \times 192$$

We know

$$(x+y)^2 = (y-x)^2 + 4xy$$

= 256 + 4 x 8 x 192

= 25 x 256

$\therefore$ x + y = 5 x 16 III

So from (ii) and (iii)

Y + x = 80

Y – x = 16

2y = 96

$\therefore$ y = 48

So,

X = 32

So, speed of slow and fast train is 32 KM/h and 48 KM/h.

Q.8 Let x is sum of the squares of any 1997 odd nos.

As x is odd, then $x^2 \equiv 1 \pmod 8$, $\Rightarrow x \equiv 1997 \pmod 8$

But, $1997 \equiv 5 \pmod 8$

$\Rightarrow x \equiv 5 \pmod 8$

As x is odd, then $x^2 \equiv 1 \pmod 8$

For example $5^2 = 25 \equiv 1 \pmod 8$

$11^2 = 121 \equiv 1 \pmod 8$

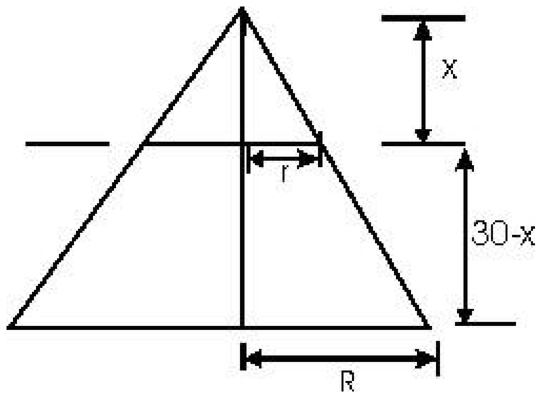
But no odd square is congruent to 5 (mod8)

So x cannot be square number

& So on.

Q. 9 from the figure $\frac{r}{R} = \frac{x}{30}$ ……. 1

From question,



$$\frac{1}{27} \text{ X } \frac{1}{3}\pi \ R^2 \text{ X } 30 = \frac{1}{3}\pi r^2 x$$

$\Rightarrow \left(\frac{r}{R}\right)^2 = \frac{30}{x} \cdot \frac{1}{27}$

$\Rightarrow x^3 = \left(\frac{30}{3}\right)^3$

$\Rightarrow x = 10$

i.e. 20 cm above the base.

Q. 10 $\left[1+\frac{1}{2^2-1}\right] \left[1+\frac{1}{3^2-1}\right] \ldots\ldots \left[1+\frac{1}{99^2-1}\right]$

$= \left[\frac{2^2-1+1}{(2-1)(2+1)}\right] \left[\frac{3^2-1+1}{(3-1)(3+1)}\right] \ldots\ldots \left[\frac{99^2-1+1}{(99-1)(99+1)}\right]$

$= \frac{2^2 . 3^2 . 4^2 \ldots . 98^2 . 99^2}{1.3.2.4.3.5 \ \ldots\ldots \ 97.99.98.100}$

$= \frac{99! \cdot 99!}{(1.2.3.4\ldots98)\ (3.4.5\ldots100)}$

$= \frac{2 . 99! . 99!}{98! \ 100!}$

$= \frac{2 \text{ x } 99}{100}$

$= \frac{99}{50}$

$= 1.98$