

14th KVS Maths Olympiad Contest – 2011

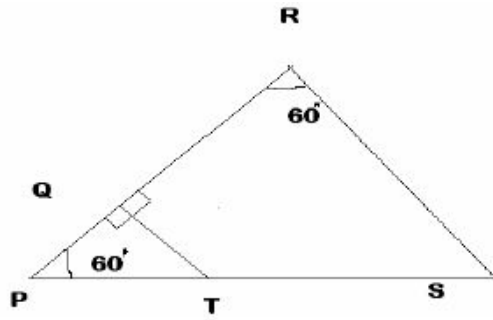
M.M. 100

Time : 3 hours

Note : (i) Please check that there are two printed pages and ten questions in all.

(ii) Attempt all questions. All questions carry equal marks

- 1) Show that for any natural number 'n' the fraction $\frac{21n+4}{14n+3}$ is in its lowest term.
- 2) a) Factorize: x^6+5x^3+8
b) Prove that $3a^4-4a^3b+b^4 \geq 0$, for all real numbers a and b.
- 3) M is any point on the minor arc BC of a circumcircle of an equilateral triangle ABC. Prove that $AM = BM+CM$.
- 4) Solve the inequality, $|x-1|+|x+1| < 4$
- 5) a) Find the square root of $\frac{3}{2}(x-1)+\sqrt{2x^2-7x-4}$, $x > 4$
b) Given real natural numbers x, y and z are such that $x+y+z = 3$,
 $x^2+y^2+z^2 = 5$, $x^3+y^3+z^3 = 7$. Find the value of $x^4+y^4+z^4$?
- 6) In the triangle given each side is of length 4 units. If the length PQ is 1 unit and TQ is perpendicular to PR, find the ratio of areas of triangle PQT and the quadrilateral QRST.



7) Prove that for any natural number 'n', the expression

$$A = 2903^n - 803^n - 464^n + 261^n \text{ is divisible by } 1897.$$

8) Find the number of odd integers between 30,000 and 80,000 in which no digit is repeated?

9) Find all the integers which are equal to 11 times the sum of their digits?

10) Prove that in any triangle ABC is one angle is 120° , the triangle formed by the feet of angle bisectors is a right angled.