

11th KVS Maths Olympiad Contest – 2008

Time : 3 Hours

M.M. : 100

NOTE: Attempt all questions. No electronic gadget is allowed during the examination.

- 1) Find the value of $S = 1^2 - 2^2 + 3^2 - 4^2 + \dots - 98^2 + 99^2$
- 2) Find the smallest multiple of '15' such that each digit of the multiple is either '0' or '8'.
- 3) At the end of year 2002. Ram was half as old as his grandfather. The sum of years in which they were born is 3854. What is the age of Ram at the end of year 2003?
- 4) Find the area of the largest square, which can be inscribed in a right angle triangle with legs '4' and '8' units.
- 5) In a Triangle the length of an altitude is 4 units and this altitude divides the opposite side in two parts in the ratio 1:8. Find the length of a segment parallel to altitude which bisects the area of the given triangle.
- 6) A number 'X' leaves the same remainder while dividing 5814, 5430, 5958. What is the largest possible value of 'X'?
- 7) A sports meet was organized for four days. On each day, half of existing total medals and one more medal was awarded. Find the number of medals awarded on each day.
- 8) Let ΔABC be isosceles with $\angle ABC = \angle ACB = 78^\circ$. Let D and E be the points on sides AB and AC respectively such that $\angle BCD = 24^\circ$ and $\angle CBE = 51^\circ$. Find the angle $\angle BED$ and justify your result.
- 9) If α , β and γ are the roots of the equation.

$$(x - a)(x - b)(x - c) + 1 = 0.$$

Then show that a, b and c are the roots of the equation

$$(\alpha - x)(\beta - x)(\gamma - x) + 1 = 0.$$

- 10) A $4 \times 4 \times 4$ wooden cube is painted so that one pair of opposite faces is blue, one pair green and one pair red. The cube is now sliced into 64 cubes of side 1 unit each.
- (i) How many of the smaller cubes have no painted face?
 - (ii) How many of the smaller cubes have exactly one painted face?
 - (iii) How many of the smaller cubes have exactly two painted faces?
 - (iv) How many of the smaller cubes have exactly three painted faces?
 - (v) How many of the smaller cubes have exactly one face painted blue and one face painted green ?