KVS Junior Mathematics Olympiad (JMO) – 2002

M.M. 100   Time : 3 hours

Note : *(i)*   *Please check that there are two printed pages and 10 questions in the question paper.*

(ii)   *Attempt all questions.*

1.   Fill in the blanks

(a) Yash is carrying 100 hundred – rupee notes, 50 fifty –rupee notes, 20 twenty – rupee notes, 10 ten –rupee notes and 5 five-rupee notes. The total amount of money he is carrying in Rupee, is ………….

(b) In a school, the ratio of boys to girls is 4:3 and the ratio of girls to teachers is 8 :1. The ratio of students to teachers is ………

(c) The value of $\left(0.5 + \dfrac{1}{0.5} \right)^{2}$ is ………….

(d) $(123456)^{2} + 123456 + 123457$ is the square of ………

(e) The area of square is 25 square centimeters. In perimeter, in centimeters, is …………….

2. (a) How many four digit numbers can be formed using the digits 1,2 only so that each of these digits is used at least once ?

(b)   Find the greatest number of four digits which when increased by 1 is exactly divisible by 2, 3, 4, 5, 6 and 7.

3. (a) If $f(x) = ax^7 + bx^5 + cx^3 - 6$, and $f(-9) = 3$, find $f(9)$.

(b) Find the value of

$$\frac{(2002)^3 - (1002)^3 - (1000)^3}{3 \times (1002) \times (1000)}$$

4.(a) If $x > 0$ and $x^4 + \dfrac{1}{x^4} = 47$, find the value of $x^3 + \dfrac{1}{x^3}$

(b) If $8^{2x} = 16^{1-2x}$, find the value of 3^{7x}.

5. A train, after traveling 70 km from a station A towards a station B, develops a fault in the engine at C, and covers the remaining journey to B at $\dfrac{3}{4}$ of its earlier speed and arrives at B 1 hour and 20 minutes late. If the fault had developed 35 km further on at D, it would have arrived 20 minutes sooner. Find the speed of the train and the distance from A to B.

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    A           C          D          B
    +-----------+----------+----------+
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6. The adjoining diagram shows a square PQRS with each side of length 10 cm. Triangle PQT is equilateral. Find the area of the triangle UQR.
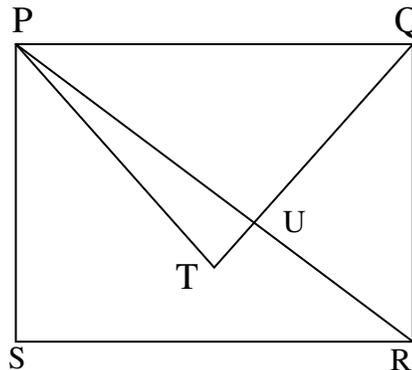
A square of side – length 64 cm is given. A second square is obtained by connecting the mid points of the sides of the first square (as shown in the diagram). If the process of forming smaller inner squares by connecting the mid points of the sides of the previous squares is continued, what will be the side-length of the eleventh square, counting the original square as the first square ?
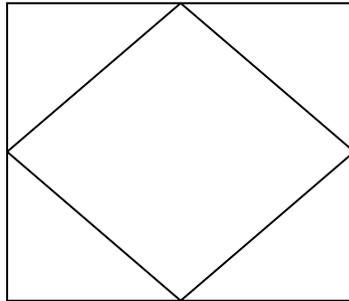
7.   Seven cubes of he same size are glued together face to face as shown in the adjoining diagram. What is the surface area, in square centimeters, of the solid if its volume is 448 cubic centimeters ?
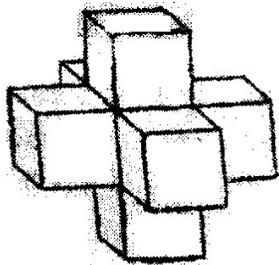
8.   Anil, Bhavna, Chintoo, Dolly and Eashwar play a game in which each is either a FOX or a RABBIT. FOXES' statements are always false and RABBITS' statements are always true.

Anil says that Bhavna is a RABBIT.

Chintoo says that Dolly is a FOX.

Eashwar says that Anil is not a FOX.

Bhavna says that Chintoo is not a RABBIT.

Dolly says that Eashwar and Anil are different kinds of animals.

How many FOXES are there ? (Justify your answer).

10.   The accompanying diagram is a road-plan of a city. All the roads go east-west or north-south, with the exception of one shown. Due to repairs one road is impassable at the point X, of all the possible routes from P to Q, there are several shortest routes. How many such shortest routes are there ?